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STANDARD VOLUME AND WEIGHT EQUATIONS FOR SITE-PREPARED LOBLOLLY PINE PLANTATIONS IN THE PIEDMONT OF SOUTH CAROLINA, GEORGIA AND ALABAMA

by

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Introduction

Loblolly pine plantations established on cut-over, mechanically prepared sites are a forest resource of increasing importance in the Piedmont region, and there is a need for reliable means of predicting yields in terms of both total and merchantable volumes, and weights, for these plantations.

During the summers of 1982 and 1983, sample trees were measured in mechanically site-prepared loblolly pine plantations in the Piedmont regions of South Carolina, Georgia and Alabama for volume and weight determination. Individual tree volume and weight prediction equations for unthinned, mechanically site-prepared loblolly pine plantations were recently developed (Clutter et al., 1983) for the Coastal Plain Regions of the Carolinas, Georgia and north Florida. The purpose of this study was to determine whether these equations could be used to predict stem volumes and weights for Piedmont loblolly, and if not, to develop new equations.

Sample Tree Data

Sample trees for this study were obtained in conjunction with the installation of permanent growth and yield plots in the Piedmont regions of South Carolina, Georgia and Alabama. Plantations sampled met the following selection criteria:

(1) at least 10 years old

(2) planted on cut-over land following mechanical site preparation

(3) unfertilized without previous thinning

(4) no evidence of excessive insect or disease damage

(5) no interplanting or excessive numbers of wildlings.

Three sample trees were felled in each plantation in which a permanent plot was installed. Two trees were selected from the dominant-codominant crown class and one from the intermediate-suppressed class. A total of 472 volume and weight sample trees were ultimately available for analysis. The distribution of these sample trees by dbh and total height classes is shown in Table 1.

Table 1--Number of felled sample trees by height and diameter classes

Diameter Class (in.)	Height Class (ft.)												
	15	20	25	30	35	40	45	50	55	60	65	70	Totals
1	2												2
2	2	10	3										15
3	1	8	28	10	4	1							52
4	1	7	17	13	16	13	4	2	1				74
5		2	9	27	15	17	7	2	1				80
6			1	17	27	17	14	9	1	1			87
7				1	10	19	20	13	6	1	1		71
8					2	9	9	15	10	3	3	1	52
9							3	8	5	4	4		24
10								3	5	2		2	12
11											1	1	2
12												1	1
Totals	6	27	58	68	74	76	57	52	29	11	9	5	472

After felling, discs one to two inches thick were removed from the trees at approximately 5-foot intervals. Field and laboratory measurements were made in order to calculate the following quantities:

(1) Total stem outside bark (ob) volume

(2) Total stem inside bark (ib) volume

(3) Merchantable ob stem volumes to outside bark diameter limits of 2", 3", 4" and 6"

(4) Merchantable ib stem volumes to outside bark diameter limits of 2", 3", 4" and 6"

(5) Total stem green weight including bark

(6) Merchantable stem green weight including bark to outside bark diameter limits of 2", 3", 4" and 6"

(7) Total stem dry wood weight (excluding bark)

(8) Merchantable stem dry wood weight to outside bark diameter limits of 2", 3", 4" and 6".

Volume of the butt bolt of each tree (ob and ib) was calculated with Newton's formula and required additional diameter measurements (ob and ib) at the midpoint of the bolt. Volumes of all other bolts were calculated with Smalian's formula while the volume of the tip was calculated as a cone.

Merchantable cubic ft. volumes (ob and ib) were calculated by determining the height to the diameter limit by linear interpolation of cross-sectional areas, except when the merchantable diameter limit was less than the diameter of the last disc, when linear interpolation of diameters was used to determine the merchantable height, and the volume was calculated as the frustum of a cone.

Green weight including bark for each stem section was calculated by assuming that within each section, the weight per cubic foot, and the cross-sectional area, vary linearly over its length, that is

$$GW = \frac{L}{6} \left[2D_1A_1 + D_1A_2 + D_2A_1 + 2D_2A_2\right]$$

where GW = green weight of the section in pounds (including bark)

D_1, D_2 = densities in pounds of green wood and bark per cubic ft. of green wood as determined for the discs at both ends

L = length of the bolt in feet

A_1, A_2 = cross-sectional areas inside bark in square ft. for the discs at both ends

Green weight (including bark) of the tip of the tree was calculated as

$$GW = D \times V$$

where D = density in pounds of green wood and bark per cubic ft. of green wood volume for the last disc

V = green volume of the tip, excluding bark

In the case of merchantable green weight, including bark, when the diameter limit fell between two adjacent discs, partial bolt weights were calculated as

$$GW = D_1A_1\ell + D_1(A_2-A_1)\frac{\ell^2}{2L} + A_1(D_2-D_1)\frac{\ell^2}{2L} + (A_2-A_1)(D_2-D_1)\frac{\ell^3}{3L^2}$$

where ℓ = distance from the bottom of the bolt to the diameter limit in ft.

Dry wood weights for the total stem and to the various merchantable diameter limits were determined by procedures similar to those used for green weight. In this case the densities were determined from the discs as dry wood weight per cubic ft. of green wood volume.

Test of Equations Developed for the Coastal Plain

The ranges of diameters and of heights of the volume and weight sample trees in the Piedmont were similar to those of the Coastal Plain sample trees. It seemed reasonable to check the suitability of the existing equations developed from the Coastal Plain sample trees, by comparing the predicted volumes and weights with the calculated volumes and weights for the Piedmont sample trees.

When the differences between predicted and calculated sample tree volumes were plotted against tree diameter and against tree height, it became apparent that the equations tended to underpredict total and merchantable stem volumes for the larger diameter and height classes. The average difference between predicted volumes and calculated sample tree volumes was positive and significantly different from zero for outside as well as inside bark stem volume as shown in Table 2. The average differences for inside bark volumes were less than for outside bark volumes, but nevertheless were also significantly different from zero for all except the 6" merchantable diameter limit.

The existing Coastal Plain weight equations tended to overpredict both the green stem weight including bark, and the dry stem wood weight as indicated in Table 2, but only the dry stem wood volumes had average differences significantly different from zero.

Table 2a--Average difference between actual and predicted weights and volumes using Coastal Plain equations.

Top Diameter Limit	Volume (cu. ft.)		Weight (lbs.)	
	O.B.	I.B.	O.B.	I.B.
6 in. top	0.1638	0.0480	-3.5277	-7.6052
4 in. top	0.2121	0.0839	-0.5622	-3.9159
3 in. top	0.1781	0.0618	-1.8025	-3.6097
2 in. top	0.1538	0.0490	-2.2343	-3.3075
Total	0.1496	0.0474	-2.3693	-3.2608

Table 2b--t-Statistics for testing the hypothesis that the average differences in Table 2a are zero.

Top Diameter Limit	Volume (cu. ft.)		Weight (lbs.)	
	O.B.	I.B.	O.B.	I.B.
6 in. top	2.53	0.87	-0.78	-4.30*
4 in. top	6.79*	3.22*	-0.22	-4.38*
3 in. top	7.32*	3.03*	-0.87	-4.98*
2 in. top	7.45*	2.81*	-1.21	-5.16*
Total	7.50*	2.82*	-1.33	-5.29*

* Significant at 0.01 level

As a result of these initial tests of the Coastal Plain equations, it was decided to fit separate volume and weight equations to the Piedmont sample tree data. The same equation forms were used and the following equations were derived:

(1) Total stem volume outside bark

$$V_{ob} = 0.0041783\ D^{1.8653}\ H^{0.9359} \qquad \text{cu. ft.}$$

In terms of $\log_e V$; $R^2 = 0.9928$ $\quad S_{y.x} = 0.0811$

(2) Total stem volume inside bark

$$V_{ib} = 0.0014793\ D^{1.8210}\ H^{1.1629} \qquad \text{cu. ft.}$$

$R^2 = 0.9898$ $\quad S_{y.x} = 0.1017$

(3) Merchantable volume outside bark

$$V_m = V_{ob}[1-0.4226\ d^{3.4580}\ D^{-3.1545}] \qquad \text{cu. ft.}$$

where d = merchantable top diameter outside bark in inches.

$R^2 = 0.9822$ $\quad S_{y.x} = 0.1689$

(4) Merchantable volume inside bark

$$V_m = V_{ib}[1-0.4482\ d^{3.5005}\ D^{-3.1947}] \qquad \text{cu. ft.}$$

$R^2 = 0.9748$ $\quad S_{y.x} = 0.2044$

(5) Total stem green weight including bark

$$GW = 0.12696\ D^{1.9159}\ H^{1.0541} \qquad \text{pounds}$$

$R^2 = 0.9818$ $\quad S_{y.x} = 0.1371$

(6) Total stem dry weight excluding bark

$$DW = 0.022323\ D^{1.7664}\ H^{1.3358} \quad \text{pounds}$$

$$R^2 = 0.9862 \qquad S_{y.x} = 0.1223$$

(7) Merchantable green weight including bark

$$GW_m = GW[1-0.5151\ d^{3.4016}\ D^{-3.1798}] \quad \text{pounds}$$

$$R^2 = 0.9693 \qquad S_{y.x} = 0.2212$$

(8) Merchantable dry weight excluding bark

$$DW_m = DW[1-0.4851\ d^{3.5745}\ D^{-3.3506}] \quad \text{pounds}$$

$$R^2 = 0.9712 \qquad S_{y.x} = 0.2251$$

(9) Taper equation

$$D_m = 1.19863\ D^{0.88415}\ H^{-0.64186}\ (H-M)^{0.68586}$$

where D_m = outside bark diameter in inches at height M feet

M = height in feet to outside bark diameter D_m inches

(10) Merchantable height equation

$$M = H - 0.76784\ D_m^{1.45803}\ D^{-1.28912}\ H^{0.93586}$$

When the Coastal Plain equations were used to predict total stem volumes and weights for the Piedmont sample trees, the sums of squared deviations were considerably higher than for the Piedmont equations as shown in Table 3.

Comparisons of volumes and weights predicted by the Piedmont and Coastal Plain equations are summarized in Table 4 and 5 respectively.

Table 3--Sums of squared deviations (natural log of volume or weight) for Coastal Plain and Piedmont equations.

Equation	Sum of Squared Deviations	
	Coastal Plain	Piedmont
Total volume outside bark	3.580	3.086
Total volume inside bark	5.284	4.847
Total green weight including bark	9.163	8.820
Total dry weight excluding bark	8.266	7.011

Table 4--Total weights and volumes produced by Coastal Plain and Piedmont equations for 5 example trees.

DBH Class (in.)	Total Height (ft.)	Total Volume (cu. ft.)			
		Outside Bark		Inside Bark	
		Coastal Plain	Piedmont	Coastal Plain	Piedmont
4	30	1.3	1.3	0.9	1.0
6	40	3.6	3.7	2.8	2.8
8	50	7.7	7.9	6.2	6.2
10	60	13.9	14.1	11.7	11.5
12	70	22.7	23.0	19.7	19.1

DBH Class (in.)	Total Height (ft.)	Total Weight (lbs.)			
		Green Weight Including Bark		Dry Weight Excluding Bark	
		Coastal Plain	Piedmont	Coastal Plain	Piedmont
4	30	67.1	65.2	25.2	24.3
6	40	197.1	192.0	76.6	73.0
8	50	432.2	421.5	172.7	163.5
10	60	802.3	783.3	327.9	309.3
12	70	1337.3	1306.7	557.4	524.4

Table 5--Merchantable weights and volumes to a 4 inch top ob produced by Coastal Plain and Piedmont equations for 4 example trees.

DBH Class (in.)	Total Height (ft.)	Merchantable Volume (cu. ft.)			
		Outside Bark		Inside Bark	
		Coastal Plain	Piedmont	Coastal Plain	Piedmont
6	40	2.9	3.1	2.2	2.3
8	50	7.1	7.3	5.7	5.7
10	60	13.4	13.6	11.3	11.0
12	70	22.2	22.5	19.3	18.7

DBH Class (in.)	Total Height (ft.)	Merchantable Weight (lbs.)			
		Green Weight Including Bark		Dry Weight Excluding Bark	
		Coastal Plain	Piedmont	Coastal Plain	Piedmont
6	40	157.8	154.9	63.6	60.6
8	50	396.5	388.9	161.4	152.9
10	60	768.9	753.5	317.7	299.8
12	70	1305.3	1278.9	548.0	515.7

Literature Cited

Clutter, J. L., William R. Harms, Graham H. Brister, and John W. Rheney. 1983. Stand structure and yields of site prepared loblolly pine plantations in the Lower Coastal Plain of the Carolinas, Georgia, and North Florida. (Prepublication draft--To be published by the USFS Southeastern For. Exp. Stn. as a Research Bulletin).