STEM VOLUME, TAPER AND WEIGHT EQUATIONS

FOR

SITE-PREPARED LOBLOLLY PINE PLANTATIONS

PLANTATION MANAGEMENT RESEARCH COOPERATIVE

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Site-Prepared Loblolly Pine Plantations

The Plantation Management Research Cooperative at the University of Georgia started the collection of a stem volume and weight sample tree data base for site-prepared loblolly pine plantations in 1977.

In 1977 data were collected from 728 felled sample trees from 182 plantations in the Lower Coastal Plain of North and South Carolina. Plantations from which these sample trees were selected had all been established after some kind of mechanical site preparation, and had not been fertilized or thinned. Four trees without any obvious stem defects were selected from each plantation: two from the larger than average dbh class and in the dominant/codominant crown classes, one of approximately average dbh, and one from the smaller dbh classes and in the intermediate/suppressed crown classes.

The Lower Coastal Plain, or Flatwoods, data base was augmented in 1981 with sample tree data from 104 additional trees obtained from 35 plantations in the Lower Coastal Plain of Georgia and north Florida. The present Flatwoods sample tree data base thus consists of 832 sample trees representing 21 counties in North and South Carolina, and 7 counties in Georgia and north Florida. Table 1.a shows the distribution of sample trees by age and site index classes, and Table 2.a shows the distribution of sample trees by dbh and total height classes.

In 1981 stem volume and weight data were also collected from 170 felled sample trees from 83 mechanically site-prepared loblolly pine plantations in the Upper Coastal Plain of Georgia. This data base was augmented in 1982 and 1983 with data from an additional 145 felled

Table 1. Distribution of Sample Trees by Age and Site Index Classes

a) Lower Coastal Plain

Age Class	Site Index Class 50	60	70	80	90	Total
6		8	7	1		16
9	15	46	98	47		206
12	10	47	83	62	2	204
15	7	31	48	50	10	146
18	4	24	51	41	10	130
21	2	6	39	8	4	59
24	2	6	42	9		59
27		6	3			9
30		3				3
Total	40	177	371	218	26	832

b) Upper Coastal Plain

Age Class	Site Index Class 40	50	60	70	80	90	Total
9		15	8	4			27
12	2	24	44	24	3	4	101
15	3	20	29	18			70
18	2	13	36	4	2		57
21	2	12	23	5	2		44
24		2	12	2			16
Total	9	86	152	57	7	4	315

c) Piedmont

Age Class	Site Index Class 40	50	60	70	80	Total
9	4	23	21	2		50
12	3	34	51	17		105
15	2	22	33	4	1	62
18		14	36	15	5	70
21		6	17	7		30
24		3	7			10
Total	9	102	165	45	6	327

Table 2. Distribution of Sample Trees by Dbh and Total Height Classes

a) Lower Coastal Plain

Dbh Class (in)	Total Height Class (feet)															
	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	Total
2		3	1													4
3	1	8	16	10	5											40
4		4	10	20	20	20	7	3								84
5		1	8	26	28	19	15	8	4							109
6				17	24	24	19	9	8	3						104
7			1	6	24	35	27	17	12	8	4					134
8					1	9	21	26	14	9	3					83
9			1			1	10	12	23	21	9	2	1			80
10						1	1	6	11	27	26	18				90
11							1	3	4	8	27	18	6		1	68
12										2	3	7	6	5		23
13											3	6	2	1		12
14													1			1
Total	1	16	37	79	102	109	101	84	76	78	75	51	16	6	1	832

b) Upper Coastal Plain

Dbh Class (in)	Total Height Class (feet)												
	15	20	25	30	35	40	45	50	55	60	65	70	Total
2	1	3	1										5
3		2	7	2									11
4	1	3	7	12	10	8	2						43
5		1	9	17	11	16	13	1	2				70
6			1	9	18	12	9	6	3				58
7					9	16	16	6	8				55
8					2	9	9	14	5	2			41
9						1	2	9	4	3			19
10									6	5		1	12
11											1		1
Total	2	9	25	40	50	62	51	36	28	10	1	1	315

c) Piedmont

Dbh Class (in)	Total Height Class (feet)												
	15	20	25	30	35	40	45	50	55	60	65	70	Total
1	2												2
2	1	7	2										10
3	1	6	21	8	4	1							41
4		4	12	9	9	9	3	2	1				49
5		1	5	18	13	14	4	2					57
6				12	20	13	12	6	1	1			65
7				1	6	11	11	10	3	1	1		44
8						3	5	10	9	3	3	1	34
9							2	5	3	2	4		16
10								3	1			2	6
11											1	1	2
12												1	1
Total	4	18	40	48	52	51	37	38	18	7	9	5	327

sample trees from 49 plantations in Georgia and Alabama. During the latter 2 years, 3 sample trees were selected from each plantation: two from the dominant/codominant crown classes, and one from the intermediate/suppressed crown classes. Presently the Upper Coastal Plain sample tree data base consists of 315 sample trees from 132 plantations in Georgia and Alabama. The distribution of sample trees by age and site index classes, and by dbh and total height classes are shown in Tables l.b and 2.b respectively.

The Piedmont sample tree data base was also collected in 1982 and 1983 from plantations in South Carolina, Georgia and Alabama. A total of 327 sample trees representing 109 mechanically site-prepared loblolly pine plantations. The distribution of this data set by age and site index classes, and by dbh and total height classes are shown in Tables l.c and 2.c respectively.

The geographic distribution of the sample tree data bases are indicated in Figure 1 for the 3 physiographic regions, as shown.

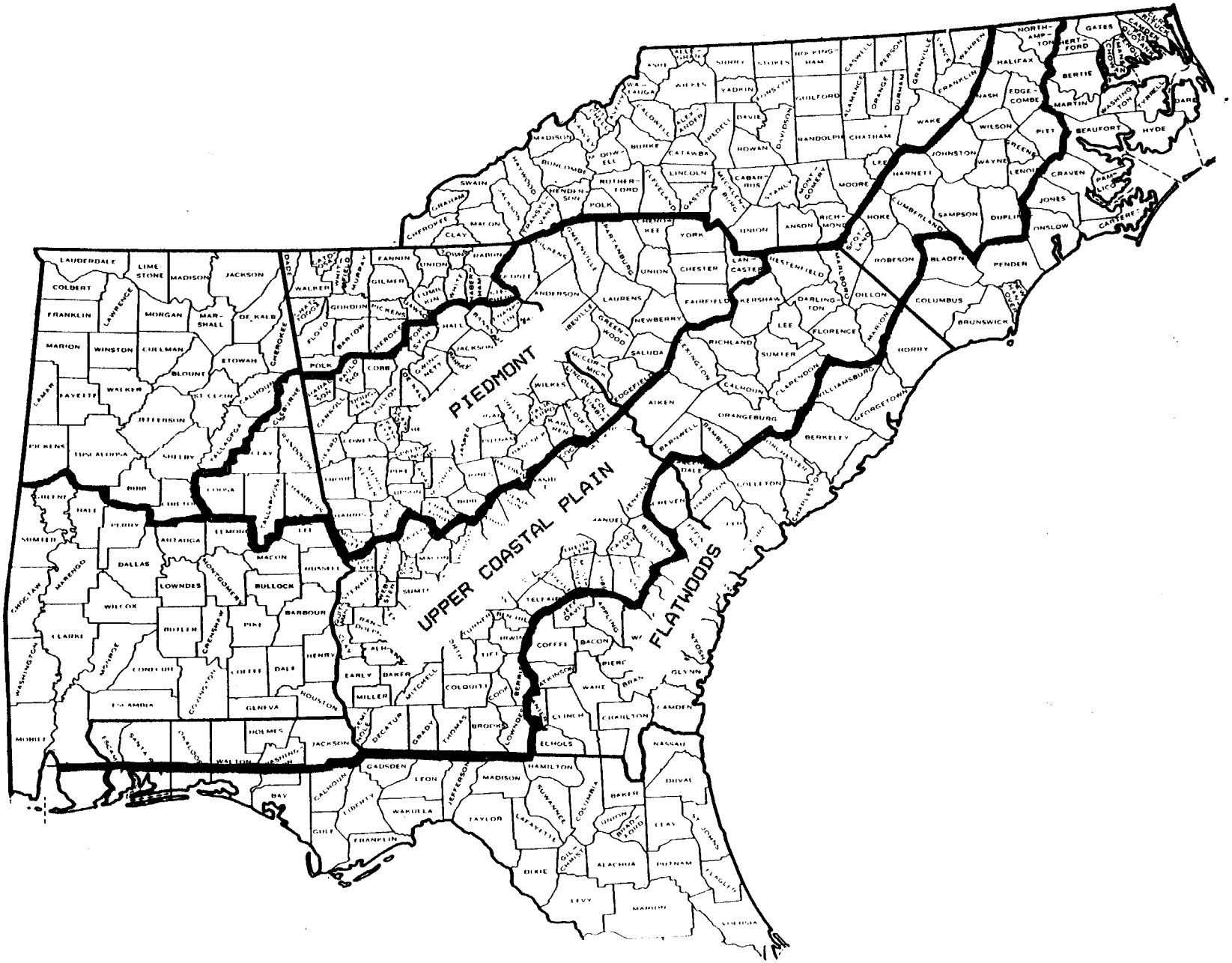
Sample Tree Measurements

Each sample tree was measured for dbh with a diameter tape to the nearest 0.1 inch, and after felling, for total height to the nearest 0.1 ft.

Trees were felled with a 0.5 ft. stump, and were cut into 5 ft. bolts to a top diameter of approximately 2 inches, except the butt bolt which was 4.5 ft. long. Mid-bolt diameters outside and inside bark were measured on the butt bolt with a diameter tape and a bark gauge to the nearest 0.1 inch.

Discs were cut from the butt end of each bolt and from the tip. Each disc was weighed to the nearest 0.1 am., and disc diameter was

Figure 1. Geographic regions as defined in PMRC sampling efforts.



measured with and without bark, with a diameter tape. The volume of each disc, green and without bark, was obtained by water displacement to the nearest 0.1 cc. All discs were then oven-dried at 102°C to a constant weight which was recorded to the nearest 0.1 gm.

Stem Volume and Weight Calculations

Cumulative stem volumes with and without bark were calculated for each sample tree to successive bolt heights, and for the total stem. Butt bolt volume was calculated as a Neiloid frustum using Newton's formula, while all other bolt volumes were calculated as paraboloid frustums using Smalian's formula, and tip volume was calculated as a cone.

Green weight with bark for each bolt was calculated as follows:

$$GW = \frac{L}{b} [2D_1A_1 + D_1A_2 + D_2A_1 + 2D_2A_2]$$

where GW = green bolt weight with bark in lbs.

L = bolt length in ft.

D_1, D_2 = density in pounds of green weight with bark per cu. ft. of green volume without bark as determined for the discs at the base and top of each bolt respectively.

A_1, A_2 = cross-sectional areas inside bark in sq. ft. for the base and top discs respectively.

Green weight with bark for the tip was calculated as the product of the green volume without bark of the tip, and the density of green wood with bark per cu. ft. of green volume without bark as determined for the disc cut from the base of the tip.

Dry weight without bark for each bolt and for the tip was calculated in a similar manner but with the respective densities

calculated from the dry disc weights without bark and green volumes without bark.

Cumulative stem green weight with bark and dry weight without bark to successive bolt heights and for the total stem were calculated for each sample tree.

Outside Bark Stem Volume and Taper Equations

Separate volume and taper equations were computed for the 3 physiographic regions. A weighted nonlinear least squares procedure with weights proportional to the inverse of volume was used to estimate parameters.

Lower Coastal Plain (Flatwoods)

$$VOB_m = 0.00145519\ D^{1.826051}\ H^{1.221965} - 0.00253872 \left(\frac{D_m^{3.741575}}{D^{1.741575}} \right) (H - 4.5)$$

where VOB_m = outside bark merchantable stem volume in cu. ft. to a top diameter limit outside bark $D_m \leqq D$.

D_m = top merchantable diameter outside bark in inches.

D = dbh in inches.

H = total tree height in feet.

$$D_m = D \left(\frac{H - M}{H - 4.5} \right)^{0.574193}$$

$$M = H - (H - 4.5) \left(\frac{D_m}{D} \right)^{1.741575}$$

where M = height above ground in feet to the outside bark merchantable diameter limit D_m in inches.

Upper Coastal Plain

$$VOB_m = 0.00431899\ D^{1.953207}\ H^{0.896934} - 0.00251744 \left(\frac{D_m^{3.714466}}{D^{1.714466}} \right) (H - 4.5)$$

$$D_m = D \left(\frac{H - M}{H - 4.5} \right)^{0.583272}$$

$$M = H - (H - 4.5) \left(\frac{D_m}{D} \right)^{1.714466}$$

Piedmont

$$VOB_m = 0.00401246\ D^{1.829011}\ H^{0.969142} - 0.00249374 \left(\frac{D_m^{3.684725}}{D^{1.684725}} \right) (H - 4.5)$$

$$D_m = D \left(\frac{H - M}{H - 4.5} \right)^{0.593569}$$

$$M = H - (H - 4.5) \left(\frac{D_m}{D} \right)^{1.684725}$$

Inside Bark Stem Volume and Taper Equations

Lower Coastal Plain (Flatwoods)

$$VIB_m = 0.00071193\ D^{1.876991}\ H^{1.321458} - 0.00217131 \left(\frac{D_m^{3.592491}}{D^{1.592491}} \right) (H - 4.5)$$

where VIB_m = inside bark merchantable stem volume in cu. ft. to a top diameter limit outside bark $D_m \leq D$.

$$D_m' = \left[0.821198\ D^2 \left(\frac{H - M}{H - 4.5} \right)^{1.062783} \right]^{0.5}$$

where D_m' = inside bark diameter in inches where the outside bark diameter is D_m inches

M = height above ground in feet to the outside bark diameter limit D_m.

Upper Coastal Plain

$$VIB_m = 0.00210741\ D^{1.957418}\ H^{1.021763} - 0.00209273\left(\frac{D_m^{3.584111}}{D^{1.584111}}\right)(H - 4.5)$$

$$D'_m = \left[0.802118\ D^2\left(\frac{H - M}{H - 4.5}\right)^{1.090512}\right]^{0.5}$$

Piedmont

$$VIB_m = 0.00171199\ D^{1.870407}\ H^{1.110322} - 0.00210729\left(\frac{D_m^{3.437603}}{D^{1.437603}}\right)(H - 4.5)$$

$$D'_m = \left[0.788358\ D^2\left(\frac{H - M}{H - 4.5}\right)^{1.040453}\right]^{0.5}$$

Green Weight With Bark

Weighted nonlinear least squares was used to estimate parameters, with weights proportional to stem weight. Separate equations were computed for the 3 physiographic regions.

Lower Coastal Plain (Flatwoods)

$$GWWB_m = 0.0740959\ D^{1.829983}\ H^{1.247669} - 0.123329\left(\frac{D_m^{3.523107}}{D^{1.449947}}\right)(H - 4.5)$$

Upper Coastal Plain

$$GWWB_m = 0.141534\ D^{1.917146}\ H^{1.038452} - 0.0932063\left(\frac{D_m^{3.589155}}{D^{1.413061}}\right)(H - 4.5)$$

Piedmont

$$GWWB_m = 0.110069\ D^{1.935455}\ H^{1.080621} - 0.0775771\left(\frac{D_m^{3.439954}}{D^{1.178473}}\right)(H - 4.5)$$

Dry Weight Without Bark

Lower Coastal Plain (Flatwoods)

$$DW_m = 0.0106276\ D^{1.882913}\ H^{1.478766} - 0.0298084\left(\frac{D_m^{3.825425}}{D^{1.517983}}\right)(H - 4.5)$$

Upper Coastal Plain

$$DW_m = 0.0290299\ D^{2.017530}\ H^{1.157743} - 0.0222220\left(\frac{D_m^{3.782287}}{D^{1.367710}}\right)(H - 4.5)$$

Piedmont

$$DW_m = 0.0360196\ D^{1.742939}\ H^{1.232462} - 0.0356069\left(\frac{D_m^{3.668307}}{D^{1.479158}}\right)(H - 4.5)$$

Dry Weight Without Bark (Known Age)

Lower Coastal Plain (Flatwoods)

$$DW_m = 0.0113113\ D^{1.901901}\ H^{1.303882}\ A^{0.210461} - 0.0309330\left(\frac{D_m^{3.821368}}{D^{1.526992}}\right)(H - 4.5)$$

Upper Coastal Plain

$$DW_m = 0.0275683\ D^{1.973518}\ H^{1.093663}\ A^{0.137418} - 0.0217837\left(\frac{D_m^{3.769104}}{D^{1.345945}}\right)(H - 4.5)$$

Piedmont

$$DW_m = 0.0288583\ D^{1.769315}\ H^{1.161088}\ A^{0.154501} - 0.0363042\left(\frac{D_m^{3.654891}}{D^{1.474768}}\right)(H - 4.5)$$

Green Weight Without Bark

Data that allowed the calculation of green weight inside bark were only available for Piedmont sample trees. In the case of the other 2 data sets, green disc weights without bark were not obtained.

Piedmont

$$GWIB_m = 0.120931\ D^{2.323008}\ H^{0.823979} - 0.076815 \left(\frac{D_m^{3.446656}}{D^{1.238789}} \right) (H - 4.5)$$

Application Example

A 20-year-old loblolly pine tree with a dbh of 10.4 inches and a total height of 66 feet has the following predicted volumes and weights, depending on the physiographic region.

	Lower Coastal Plain	Upper Coastal Plain	Piedmont
VOB	17.5 ft^3	17.9	16.9
VOB_6	15.4 "	15.8	14.7
VOB_4	17.0 "	17.4	16.4
VIB	14.7 "	14.9	14.3
VIB_6	12.6 "	13.0	12.2
VIB_4	14.2 "	14.5	13.8
GWWB	1002.6 lb	977.6	946.9
$GWWB_6$	862.4 "	847.6	803.4
$GWWB_4$	969.0 "	947.3	911.4
DW	428.6	418.1	373.0
DW_6	379.0	369.4	323.9
DW_4	418.1	407.6	361.9
$D_{16.8}$	9.1"	9.1"	9.1"
$D'_{16.8}$	8.4"	8.2"	8.2"
$M_{6"ob}$	42.4'	42.0'	41.7'
$M_{6"ib}$	39.7'	38.5'	39.1'