PMRC YIELD PREDICTION SYSTEM

FOR SLASH PINE PLANTATIONS

IN THE ATLANTIC COAST FLATWOODS

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Yield Prediction for Slash Pine Plantations in the Atlantic Coast Flatwoods

This report summarizes results of some of the growth and yield research undertaken by the Plantation Management Research Cooperative[1] (PMRC) at the University of Georgia during the past 15 years.

The yield prediction system presented here was developed for mechanically site-prepared slash pine plantations in the Atlantic Coast Flatwoods. It is based on three data sets: One data set consists of 683 temporary growth and yield sample plots and 206 monumented plots with a 4-year remeasurement. These plots were established in unthinned and unfertilized plantations that were at least 10 years old. Plots were rectangular in shape, designed to include approximately 64 original planting spaces, thus varying in size. Average plot size was approximately one tenth of an acre. The geographic distribution and a summary of the range of ages, site qualities and stand densities of these plots can be found in PMRC Technical Report 1988-1.

A second data set consists of 838 felled sample trees from more than 200 plantations in 36 counties in Georgia and north Florida. A description of sample tree measurements and calculations of stem volumes and weights, as well as the distribution of sample trees by dbh and total height classes can be found in PMRC Technical Report 1988-1. This data set was used to derive simultaneous total and merchantable stem volume and compatible taper equations for outside and inside bark stem volume and taper. Total and merchantable stem weight prediction equations for green weight with bark, green wood weight, and dry wood

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[1]Current members of the PMRC who support this work are Champion International Corp., Container Corp. of America, Georgia Pacific Corp., Gilman Paper Co., Inland Rome Inc., ITT Rayonier Inc., Kimberly Clark Corp., MacMillan Bloedel Inc., Nekossa Packaging Inc., Procter & Gamble Cellulose, St. Joseph Land & Development Co., and Union Camp Corp.

weight can also be found in PMRC Technical Report 1988-1.

The third data set is from a spacing and thinning study installed during the 1960's at 114 locations in the Flatwoods in 2- and 3-year-old mechanically site-prepared and unfertilized plantations. Planting survival densities of 100, 200, 300, 450 and 700 were represented at each location, and 900 trees per acre at some locations, with as many as 7 measurements at 3-year intervals since age 5. At 29 installations additional plots were available, representing different planting densities, and these plots were thinned once in 1978 and 1980 with different thinning intensities at ages ranging from 10 to 17 years as explained in PMRC Technical Report 1989-4. Thinned plots have been measured on a 2-year cycle since thinning with the last measurement in 1988.

Yield prediction models are useful tools in making timber management decisions. Three separate sections of this report contain yield prediction models with different input requirements that can be used for this purpose under different circumstances.

The type of yield prediction model presented in Section 1 is most appropriate for the evaluation of proposed management regimes for future unthinned plantations. The only required input is the site index, the planting survival density and the age. It provides an analytical framework to address the important decisions of how many trees to plant and when to harvest the crop, given the product goal and associated prices and silvicultural costs. More general models of this type will be necessary in the future to accommodate different silvicultural practices.

The methodology presented in Section 2 deals with the problem of predicting the per-acre yield of existing plantations and the projection of future yields for management planning purposes. Required minimum input information are the age, the surviving number of trees per acre, the average dominant/codominant height, and for thinned plantations, the age when thinned and the percentage of the trees removed in the thinning.

Section 3 is a stand-table-based yield prediction system. Required input is a stand table. In the case of a previously unthinned plantation, a Weibull-based stand table can be predicted from the age, surviving trees per acre and average dominant/codominant height. In either case the current stand table provides the basis for the current yield prediction, and a stand table projection procedure is used to predict future stand and stock tables.

1. A Stand Level Growth Model for Unthinned Plantations

Classical growth models can be used to describe the growth of unthinnined plantations when longterm remeasurement data are available. The plantation spacing study installed during the 1960's at 114 locations provided this kind of data. Individual study plots had been measured as many as 7 times at 3 year intervals since age 5, and these remeasurement data were used to estimate the parameters in the following growth models:

$$B = 2.041\ S^{1.127}\{1 - 1.108\ \exp[(-0.015 - 0.000069\ TP^{0.950})A]\}^{1.208} \tag{1.1}$$

$$V = 0.043\ S^{1.70}[1 - 1.058\exp\{(-0.0082\ TP^{0.349})A\}]^{3.187} \tag{1.2}$$

where

B = per-acre basal area in sq. ft.

V = per-acre inside bark stem volume in cunits

S = base-age 25 site index (equation 2.5)

TP = planting survival at age 2, as trees per acre.

A = plantation age in years.

A growth model for any specific site index and age 2 planting survival can readily be obtained from equation (1.2.). For example, for site index 70 and an age 2 survival of 700 trees per acre

$$V = 58.90\ [1 - 1.058\ \exp(-0.0807A)]^{3.187} \tag{1.3}$$

Per-acre volume growth curves generated with equation 1.2 for site index 70 and age 2 planting survival densities of 300, 500, 700 and 900 trees per acre are shown in Figure 1.

The age when the mean annual total stem volume growth (MAI) reaches a maximum is obtained by classical optimization procedure. For example, for the growth curve of equation (1.3), by solving the

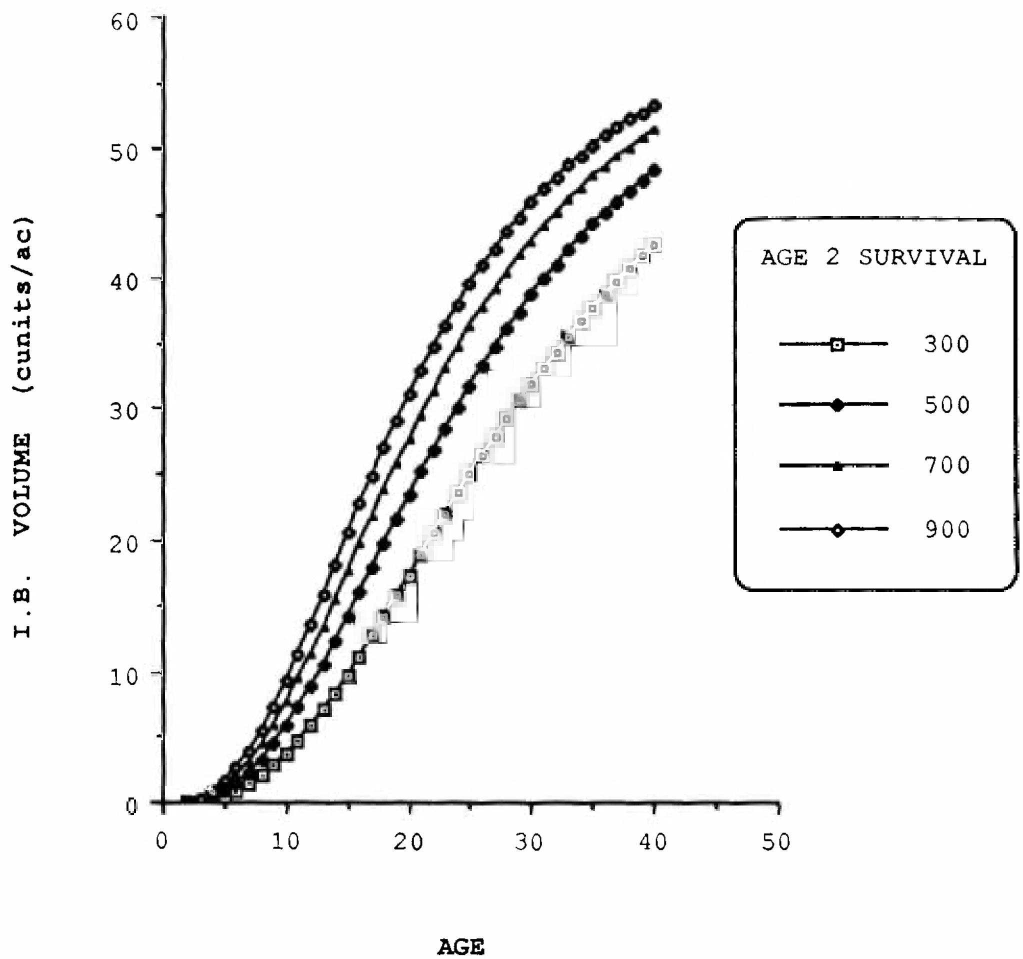


Figure 1. Per-acre volume growth curves for site index 70 and age 2 densities of 300, 500, 700, and 900 trees per acre.

following equation for age A.

$$58.90(1.058)\exp(-0.0807A)\ [0.0807(3.187)A + 1] - 58.90 = 0$$

when A = 26 years (solved iteratively)

For a single product, such as pulpwood, and a constant price per unit, P, the harvest age that maximizes the bare land value (BLV) can also be readily obtained, with the BLV defined as

$$BLV = \frac{P.V - R.e^{\delta A}}{e^{\delta A}-1} - \frac{T}{e^{\delta} - 1}$$

where

R = plantation establishment cost per acre

δ = instantaneous equivalent of the annual discount rate i,

that is, $\delta = \ell n\ (1 + i)$

T = annual ad valorem tax per acre.

Using the parameters of equation (1.3), the age that maximizes the BLV is found by solving the following equation for A

$$P(58.90)\delta[e^{\delta A}-1.058e^{(\delta-0.0807)A}] + P(58.90)(1.058)(0.0807)(3.187)[e^{-0.0807A} - e^{(\delta-0.0807)A}]$$

$$-R\delta e^{\delta A}(1 - 1.058e^{-0.0807A})^{1-3.187} = 0$$

For P = \$25/cunit, R = \$150/acre, i = 0.05 so that $\delta = 0.04879$

A = 21 years (solved iteratively)

Harvest ages that will maximize total volume production or bare land value for other planting survival densities, discount rates, establishment costs and prices can be found in a similar manner with the appropriate set of parameter values.

The predicted total per-acre volume can be apportioned into various product categories such as pulpwood, chip-n-saw volume, and large sawtimber volume. For this purpose, a per-acre merchantable volume prediction equation proposed by Amateis and others* (1986) was fitted to the PMRC growth and yield plot data.

$$V_{d,t} = V \exp[-0.52(t/\bar{D})^{3.84} - 0.69N^{-0.12}(d/\bar{D})^{5.72}] \qquad (1.4)$$

where

V = total per-acre inside bark volume in cunits

$V_{d,t}$ = merchantable per-acre volume in cunits to a t" top diameter outside bark for trees with dbh ≥ d"

$\bar{D}$ = quadratic mean dbh in inches

N = surviving trees per acre

For example, according to equation (1.3), a plantation with site index 70 and age 2 survival of 700 trees per acre is expected to yield a total inside bark stem volume at age 30 of

$$V = 43.00 \text{ cunits per acre}$$

A survival equation predicts a survival of 400 trees at age 30 (see equation 2.3), and equation 1.1 predicts a basal area of 173.5 sq.ft. per acre, so that $\bar{D}$ = 8.9". Equation 1.4 can now be used to apportion the total volume into desired product classes that may be defined as follows:

Large sawtimber volume: $V_{10,8}$ = 15.98 cunits per acre

Chip-n-saw volume: $V_{8,6} - V_{10,8}$ = 32.04 - 15.98 = 16.06 cunits per acre

Pulpwood volume: $V_{4,2} - V_{8,6}$ = 42.78 - 32.04 = 10.74 cunits per acre

The expected development over time of these 3 product class volumes for this plantation is shown in Figure 2.

*Amateis, R.L., H.E. Burkhart, T.E. Burk. 1986. A ratio approach to predicting merchantable yields of unthinned loblolly pine plantations. For. Sci. 32(2): 287-296.

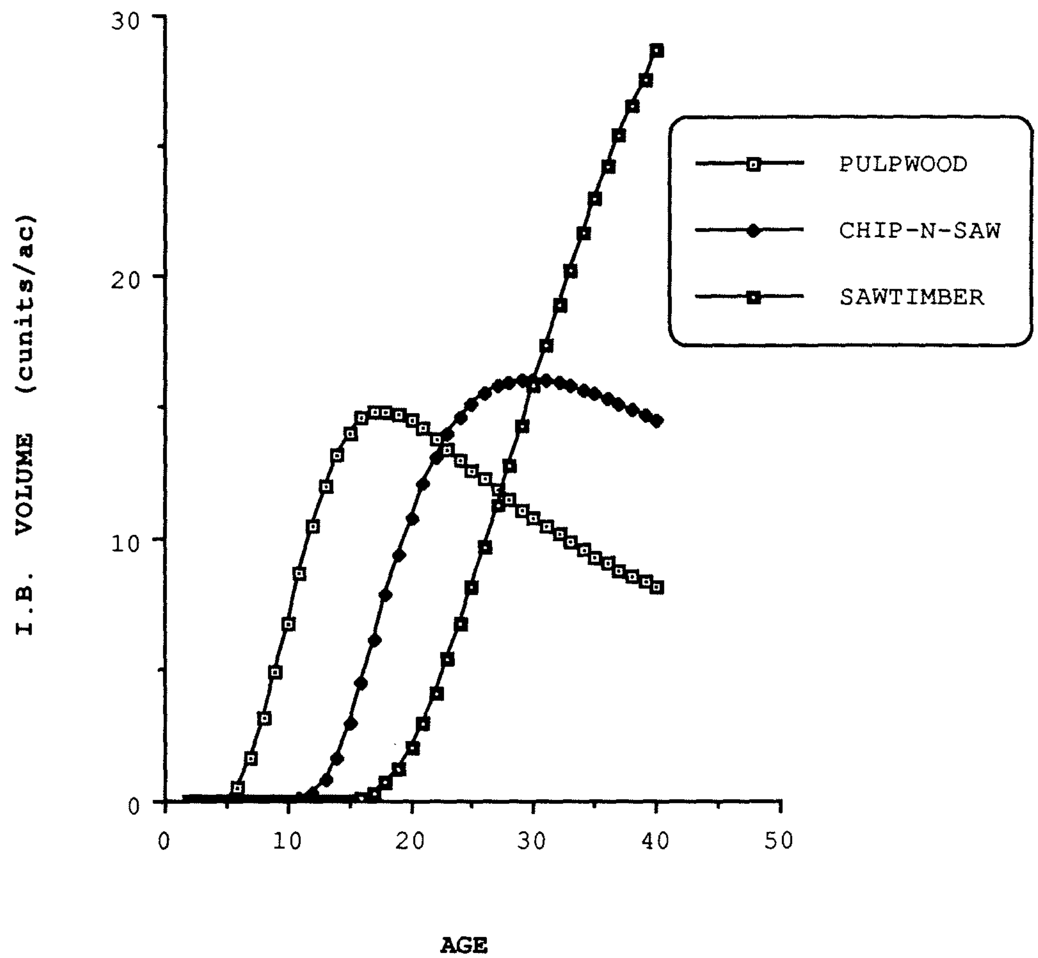


Figure 2. Per-acre volume growth curves for pulpwood, chip-n-saw, and large sawtimber.

2. Stand Level Yield Prediction and Projection

The following general conclusions are based on data obtained from the PMRC growth and yield plots, and from the spacing and thinning study plots.

Over the range of stocking levels and ages represented in these studies, plots of the same age and with the same number of surviving trees per acre generally produced more basal area and total stem volume, the higher the average dominant/codominant height. At a given location and age, more surviving trees produced more basal area and more total volume. Thinned plots with a given number of trees per acre had less basal area and total volume per acre than neighboring unthinned plots of the same age with the same number of trees per acre. The greater the thinning intensity, the greater the difference tended to be.

2.1 Per-acre Basal Area Prediction and Projection

A per-acre basal area prediction and compatible projection equation that account for these trends were derived from the two data bases.

$$\ell n\ B = -4.8066 - 26.2731(1/A) + 1.5116\ \ell n\ H + 0.5270\ \ell n\ N + 4.1293\ (\ell n\ H)/A + 2.4966(\ell n\ N)/A - 0.0832\ (N_t/N_b)(A_t/A) \tag{2.1}$$

where

B = per-acre basal area in sq.ft.

A = plantation age in years

H = Average dominant/codominant height in ft. at age A

N = surviving trees per acre at age A

N_t = trees per acre removed in thinning at age $A_t \le A$

N_b = surviving trees per acre immediately before thinning at age A_t

A_t = age when thinning occurred

ℓn = natural logarithm

$$\ell n\, B_2 = A_1/A_2\, \ell n\, B_1 - 4.8066\,(1 - A_1/A_2) + 1.5116\,(\ell n\, H_2 - A_1/A_2\, \ell n\, H_1) + 0.5270\,(\ell n\, N_2 - A_1/A_2\, \ell n\, N_1) + 4.1293\,(\ell n\, H_2 - \ell n\, H_1)/A_2 + 2.4966\,(\ell N_2 - \ell n\, N_1)/A_2 \quad (2.2$$

where B_1 and B_2, H_1 and H_2, N_1 and N_2 are the basal areas per acre, the average dominant/codominant heights and the number of surviving trees per acre at ages A_1 and A_2 respectively, and $A_t \leq A_1 \leq A_2$. The average absolute difference between actual and predicted basal area is 10% of actual basal area.

Predicting future basal area with either equation 2.1 or equation 2.2 requires projections of survival and of average dominant/codominant height.

2.2 Survival Projection

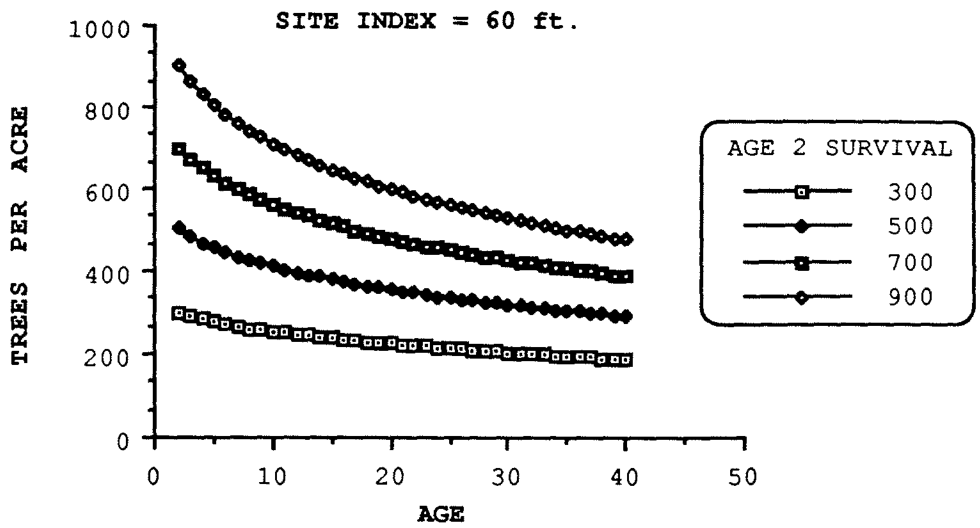
In plantations the survival rate due to intraspecific competition on a given site depends on both the number of survivors and the age. The survival rate at a given age and density is often also affected by site quality. A modified form of a survival function proposed by Clutter and Jones* (1980) accounts for such effects.

$$N_2 = \{N_1^{-0.3008} + (0.03272 - 0.9628/S)\,[(A_2/10)^{0.5245} - (A_1/10)^{0.5245}]\}^{-3.3245} \quad (2.3)$$

where N_1 and N_2 are the numbers of surviving trees per acre at ages A_1 and A_2 respectively, and S is the site index (see equation 2.5).

Equation 2.3 was used to generate the survival curves shown in Figure 3 for age 2 planting survival densities of 900, 700, 500 and 300 trees per acre, for site indexes 60 and 70.

*Clutter, J.L. and E.P. Jones. 1980. Prediction of growth after thinning in old-field slash pine plantations. USDA For. Serv. Research Paper SE-217.



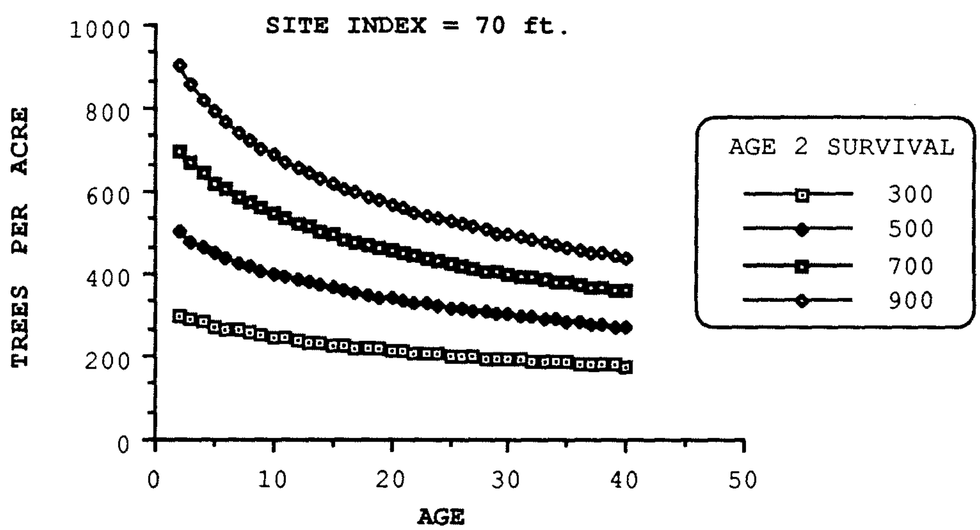


Figure 3. Predicted survival for age 2 densities of 300, 500, 700, and 900 trees per acre, and site indeces of 60 and 70 feet.

2.3 Average Dominant/Codominant Height Projection

The following projection equation was fitted to the remeasured plot data for the average height of all trees classified as dominant or codominant.

$$H_2 = H_1 \{[1-\exp(-0.0456A_2)]/[1-\exp(-0.0456A_1)]\}^{1.183} \tag{2.4}$$

where H_1 and H_2 are the average dominant/codominant heights at ages A_1 and A_2 respectively. The implied site index equation with an index age of 25 years is

$$H = 1.5776\ S[1-\exp(-0.0456A)]^{1.183} \tag{2.5}$$

where S is the site index. Equation 2.5 was used to generate the site index curves in Figure 4.

2.4 Per-acre Volume Prediction and Projection

Per-acre volume yield predictions in plantations are often based on age, a measure of stand density such as the number of surviving trees per acre and/or basal area per acre, and a measure of site quality such as the site index or the average height of trees in the dominant canopy.

A simultaneous estimation procedure was used to estimate the parameters in a total stem volume prediction equation and a compatible projection equation with age, predicted per-acre basal area and projected survival and average dominant/codominant height as predictor variables in the projection equation 2.7.

$$\ell n\ V = 2.9939 - 1.1497\ (\ell n\ H)/A - 0.3371\ \ell n\ N + 1.4813\ \ell n\ B \tag{2.6}$$

where

V = per-acre total inside bark stem volume in cu.ft.

H = average dominant/codominant height in ft.

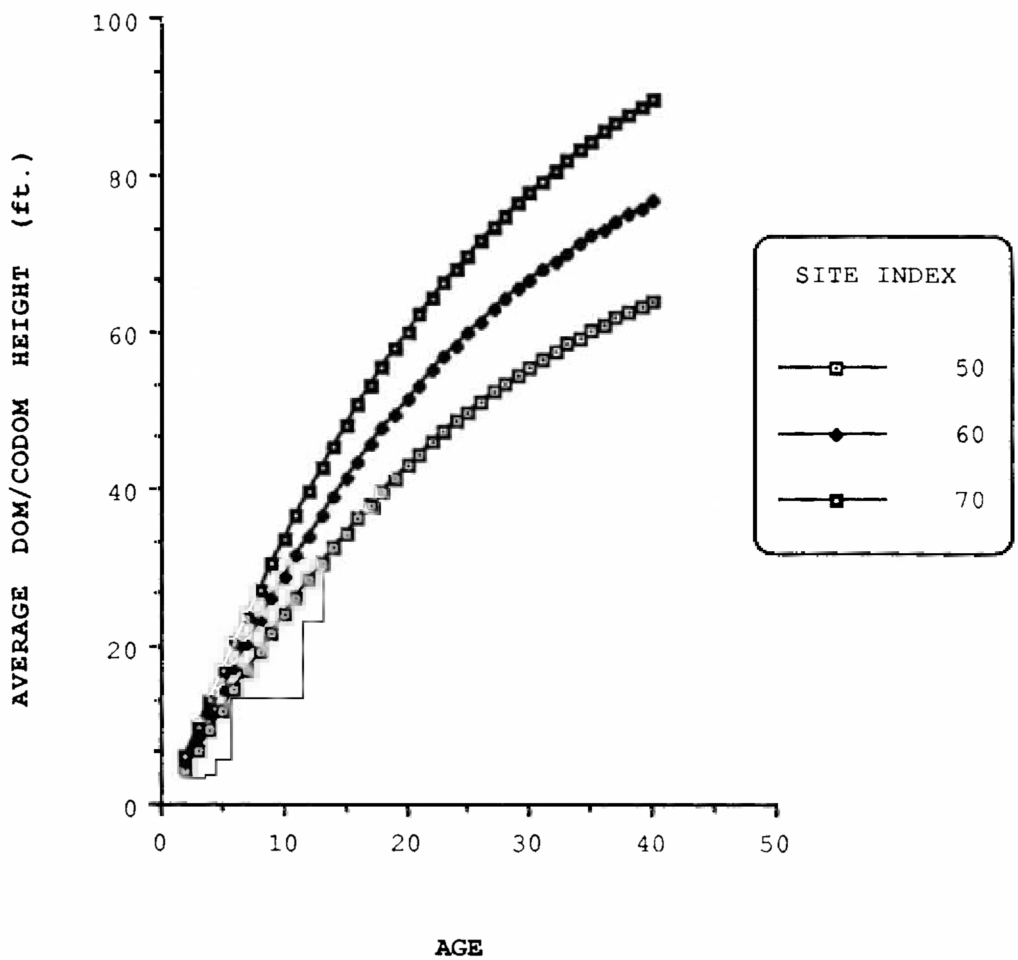


Figure 4. Dominant / codominant height growth curves for site indeces of 50, 60, and 70 feet.

A = plantation age in years

N = surviving trees per acre

B = per-acre basal area in sq. ft.

$$\ell n\ V_2 = \ell n\ V_1 - 1.1497(\ell n\ H_2/A_2 - \ell n\ H_1/A_1) - 0.3371\ (\ell n\ N_2 - \ell n\ N_1) + 1.4813(\ell n\ B_2 - \ell n\ B_1) \quad (2.7)$$

where V_1 and V_2, H_1 and H_2, N_1 and N_2, and B_1 and B_2 are the total stem volumes, average dominant/codominant heights, surviving trees per acre, and basal areas per acre at ages A_1 and A_2 respectively.

A common volume prediction applies to both unthinned and thinned stands. In the PMRC thinning study, as in many others, it seems clear that management regimes that include thinning will not produce as much total stem volume over realistic rotations as will regimes without thinnings. Justification for thinning must therefore be sought in different product flows and cash flows. For this purpose the merchantable volume prediction equation 1.4 can be used.

The following example serves to illustrate the calculations to predict the yield for a plantation that is currently 15 years old with 495 trees per acre and an average height of dominants/codominants of 48', and to project the expected yield to age 30.

Current Plantation		Projected Plantation	
Age = 15 years	(given)	Age = 30 years	(given)
H = 48 ft	(given)	H = 78 ft	(eq. 2.4)
N = 495/ac	(given)	N = 400/ac	(eq. 2.3)
B = 106 ft^2/ac	(eq. 2.1)	B = 174 ft^2/ac	(eq. 2.1 or 2.2)
V = 1832 ft^3/ac	(eq. 2.6)	V = 4672 ft^3/ac	(eq. 2.6 or 2.7)
Pulpwood 1462 ft^3/ac	(eq. 1.4)	Pulpwood 1160 ft^3/ac	(eq. 1.4)
C-n-s 313 ft^3/ac	(eq. 1.4)	C-n-s 1740 ft^3/ac	(eq. 1.4)
Sawtimber none	(eq. 1.4)	Sawtimber 1748 ft^3/ac	(eq. 1.4)

A plantation with an age 2 planting survival of 700 per acre and a site index of 70, is expected to have 495 trees surviving at age 15 according to equation 2.3 and an average dominant/codominant height of

48′ according to equation 2.5. These two equations, together with equations 2.1 and 2.6 can be used to construct per-acre basal area and total stem volume yield curves shown in Figures 5 and 6, and the development of pulpwood volume, chip-n-saw volume, and large sawtimber over time can be obtained with equation 1.4 as shown in Figure 7. Implied yield curves for other unthinned plantations with different planting survival densities and site indexes can be constructed in a similar manner.

2.5 Per-Acre Yield Prediction for Thinned Plantations

When plantations are thinned selectively from below as in the PMRC thinning study, the thinning is most commonly specified in terms of the number of trees per acre to remain after thinning, with the thinning intensity expressed as the percentage of the trees that are removed in the thinning. The percentage of the trees removed can be converted to a percentage of the basal area removed as follows

$$(B_t/B) = (N_t/N)^{1.2248}$$

where B and N are the basal area and trees per acre before thinning, and B_t and N_t the basal area and trees per acre removed in thinning. For row thinnings the exponent in equation 2.8 is 1.0, and for combination row x selective thinnings the exponent takes on intermediate values depending on the relative amounts of row and selective removals. For example, in a separate thinning study where 34% to 44% of the thinned trees were removed selectively from below, and the complement in row thinning, the estimated exponent was 1.1351. In each case, the percentage of the total stem volume removed was approximately equal to the percentage of basal area removed.

Suppose the 15-year-old plantation of the previous example with 495 trees and 106 sq.ft. of basal area per acre, and an average

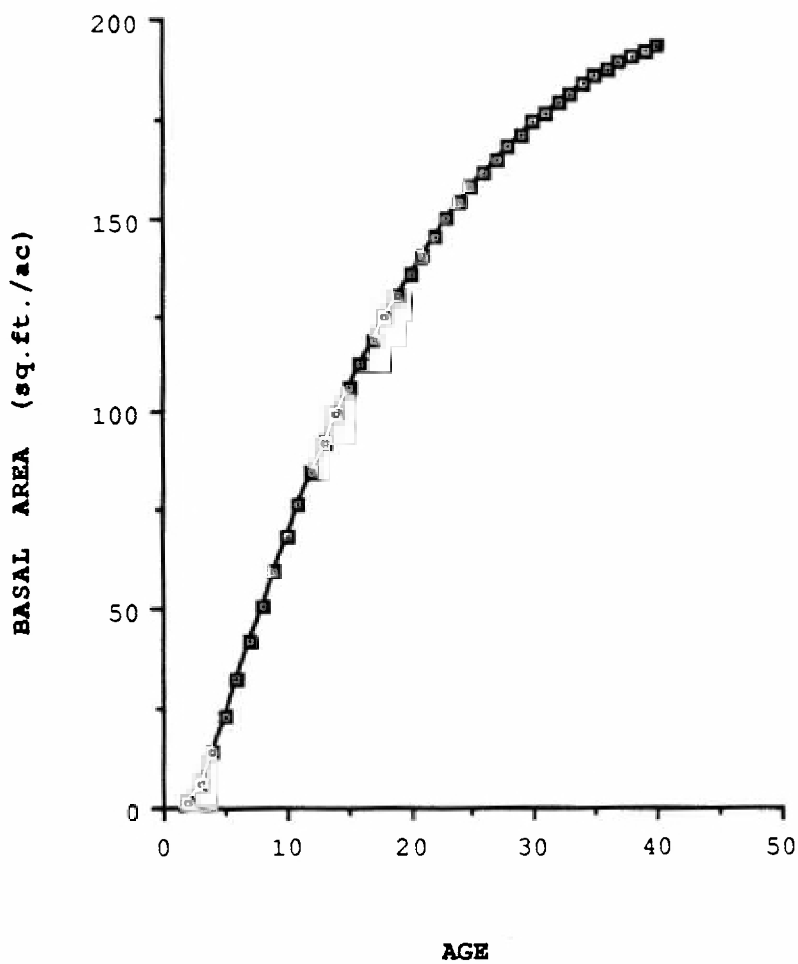


Figure 5. Predicted per-acre basal area growth curve.

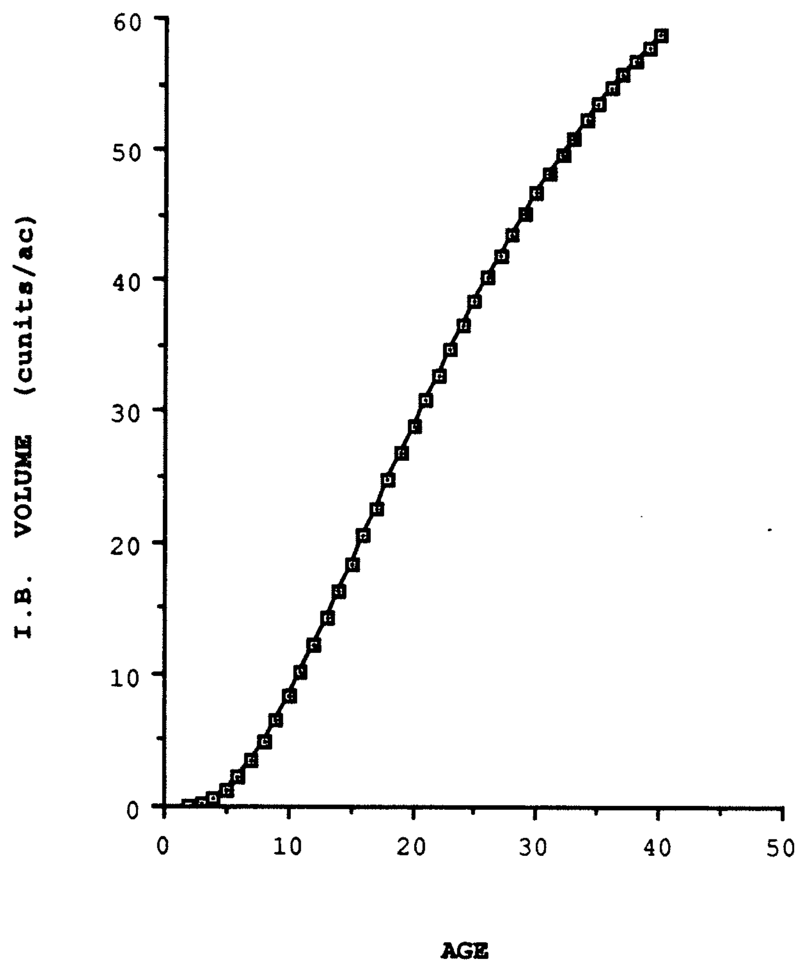


Figure 6. Predicted per-acre volume growth curve.

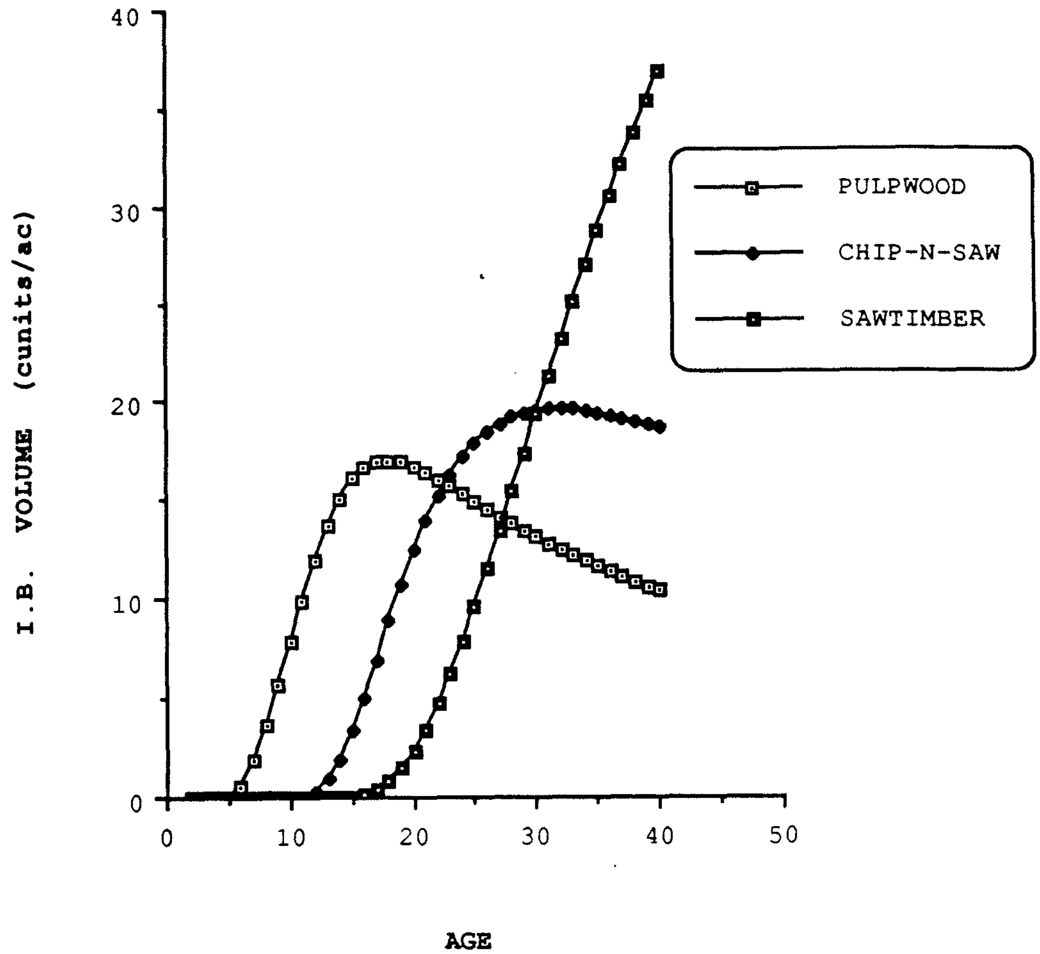


Figure 7. Predicted per-acre volume growth curve for pulpwood, chip-n-saw, and large sawtimber.

dominant/codominant height of 48', is thinned selectively from below to 300 trees. Then the basal area removed is calculated as

$$B_t = 106(195/495)^{1.2248}$$

$$= 34 \text{ sq. ft. per acre}$$

And the total volume removed in thinning is

$$V_t = 1832(34/106)$$

$$= 588 \text{ cu. ft. per acre}$$

Equation 1.4 can be used to apportion this volume, giving a total merchantable volume of 554 cu. ft., mostly pulpwood with a negligible amount of small sawtimber.

After thinning there will be 300 trees left with 72 sq. ft. of basal area and 1244 cu. ft. per acre. When a final harvest is planned at age 30, the expected yield can be projected as follows. At age 30

$H = 78$ ft (eq. 2.4)

$N = 250$/acre (eq. 2.3)

$B = 128$ ft^2acre (eq. 2.2)

$V = 3473$ ft^3/acre (eq. 2.7)

Pulpwood 618 ft^3/acre (eq. 1.4)

Chip-n-saw 1068 ft^3/acre (eq. 1.4)

large sawtimber 1775 ft^3/acre (eq. 1.4)

Figure 8 shows the implied total volume yield curves for an unthinned plantation with an age 2 survival of 700 trees per acre and a site index of 70, and for an age 2 survival of 403 trees per acre with an expected survival of 300 at age 15. Also shown in Figure 8 is the expected yield curve for the thinned plantation with an age 2 survival of 700, thinned to 300 at age 15.

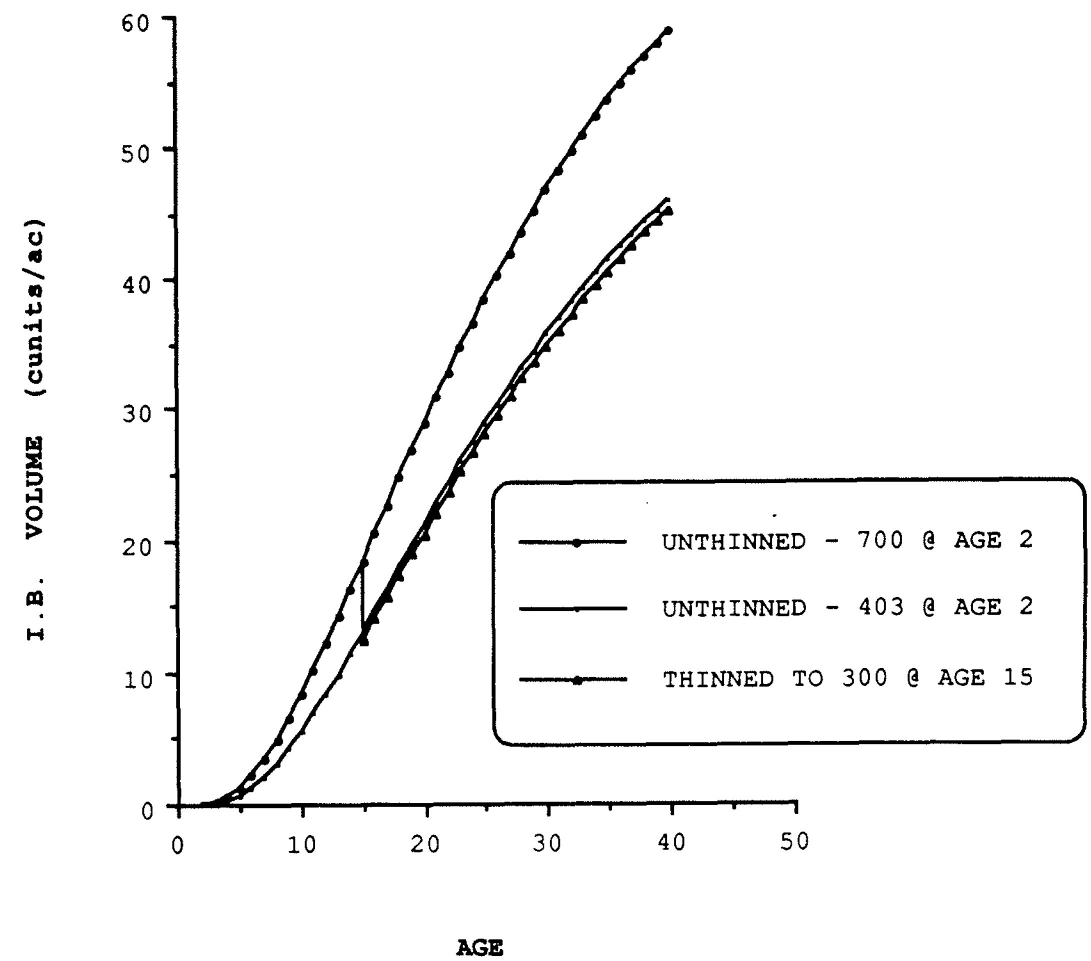


Figure 8. Predicted per-acre volume growth curves for comparable thinned and unthinned plantations.

Similar yield calculations can be performed for row thinnings by using an exponent of 1.0 instead of 1.2248 in equation 2.8, and for combination row x selective thinnings with an appropriate exponent.

3. Stand and Stock Table Prediction and Projection

If a stand table for an unthinned plantation is not available, one can be predicted from commonly available stand level variables. Certain percentiles of the dbh distribution are predicted from which the parameters of an assumed Weibull distribution can be recovered. For example, for the 15-year-old plantation with 495 surviving trees, 106 sq. ft. of basal area per acre, and an average dominant/codominant height of 48 ft, the 0^{th}, 24^{th}, and 93^{rd} percentiles are predicted as

$$\ell n\ X_o = 2.7321 - 0.1401\ \ell nN + 0.6418\ \ell n(B/N) \tag{3.1}$$

$$X_o = 2.4''$$

where X_o = the smallest dbh in inches

N = surviving trees per acre

B = per-acre basal area in sq ft.

ℓn = natural logarithm.

$$\ell n\ X_{24} = 4.0320 - 0.4244\ \ell nH + 0.0725\ \ell n\ N + 0.7756\ \ell n(B/N) \tag{3.2}$$

$$X_{24} = 5.2''$$

where X_{24} = the 24^{th} percentile in inches

H = average dominant/codominant height ft.

$$\ell n\ X_{93} = 1.7749 + 0.2902\ \ell n\ H - 0.0572\ \ell n\ N + 0.2991\ \ell n\ (B/N) \tag{3.3}$$

$$X_{93} = 8.0''$$

For the Weibull distribution

$$F(D) = 1 - \exp[-((D-a)/b)^{c}] \quad , \quad a \le D \le \infty$$

where

$F(D)$ = probability that dbh $\le$ D"

a,b,c = parameters defining the distribution

The parameters of the distribution are obtained as follows

$$a = X_o - 1, \text{ if } x_o > 2"$$

$$= X_o/2, \text{ if } x_o \leq 2"$$

$$= 1.4"$$

$$c = 2.2711/[\ell n(x_{93} - a) - \ell n\ (x_{24} - a)]$$

$$= 4.114$$

$$b = -a\Gamma_1/\Gamma_2 + [(a/\Gamma_2)^2\ (\Gamma_1^2 - \Gamma_2) + \bar{D}^2/\Gamma_2]^{0.5}$$

$$\textit{where} \quad \Gamma_1 = \Gamma(1+1/c) = \Gamma(1.2431) = 0.9079$$

$$\Gamma_2 = \Gamma(1 + 2/c) = \Gamma(1.4861) = 0.8859$$

$\Gamma(\ \ ) =$ Gamma function

$\bar{D} =$ quadratic mean dbh in inches

So that $b = 5.211$

The number of trees in a dbh class with a lower limit of D_L and an upper limit of D_u is obtained as

$$N[F(D_u) - F(D_L)],$$

bearing in mind that $a \leq D \leq \infty$. The following stand table is obtained for the 15-year-old plantation with N=495, H=48′ and B = 106 sq. ft. per acre.

Table 1. Predicted stand and stock table for a 15-yr-old plantation with N = 495, H = 48', B = 106 sq. ft./ac.

Dbh-Class in.	Tree/ac	H_i ft.	B_i/ac sq. ft.	V_i^* (2" top)/ac cu.ft.
2	1	19.2	0.02	-
3	11	29.3	0.53	4.9
4	43	36.5	3.79	50.0
5	99	41.8	13.49	210.3
6	143	45.5	28.02	482.0
7	125	48.2	33.42	611.9
8	59	50.2	20.68	394.6
9	13	51.6	5.69	111.6
10	1	52.6	0.55	11.4
	495		106.19	1876.7

* inside bark volume

The stock table is obtained from a predicted average total tree height, H_i, for each dbh class midpoint D_i

$$H_i = 1.150\ H\ [1-1.257\ \exp(-2.058(D_i/\bar{D}))] \tag{3.4}$$

where

H_i = average total height in ft. for trees with dbh = D_i"

H = average dominant/codominant height in ft.

D_i = dbh - class midpoint in inches

$\bar{D}$ = quadratic mean dbh in inches

and a simultaneous total and merchantable inside bark volume equation

$$V_i = 0.002253\ D_i^{1.9941}\ H_i^{0.9834} - 0.002028(D_m^{3.6794}/D_i^{1.6794})(H_i - 4.5) \tag{3.5}$$

where

V_i = inside bark stem volume in cu. ft.

D_i = tree dbh in inches

H_i = total tree height in ft.

D_m = merchantable diameter limit outside bark in inches.

A similar standard volume equation for total and merchantable stem volume with bark, and weight prediction equations for green weight with bark as well as dry wood weight, are given in PMRC Technical Report 1988-1.

3.1 Stand Table Projection

In many instances an initial per-acre stand table will be available from a management inventory, or can be predicted as described above, and the objective is to derive a future stand table. A stand table projection procedure described by Clutter and Jones (1980) is used for this purpose. In this procedure prior estimates of survival and of future per-acre basal area are assumed to be available.

The procedure is based on an assumption about the change in relative tree size over time. If b_{1i} and b_{2i} are the basal areas of the i^{th} surviving tree at ages A_1 and A_2 respectively, it is assumed that

$$(b_{2i}/\bar{b}_2) = (b_{1i}/\bar{b}_1)^{(A_2/A_1)^c}$$

where $\bar{b}_1$ and $\bar{b}_2$ are the average basal areas of survivors at age A_1 and A_2, and c is a parameter estimated from remeasurement data of individually identified trees. An estimate of the parameter, c, was based on individually identified sample trees representative of the range of diameters on each of 254 remeasured sample plots. A least squares estimate of c = 0.05968 was obtained with a standard error of 0.008.

A projected stand table consistent with the projected per-acre basal area is obtained as follows

$$n_i b_{2i} = n\,\bar{b}_2\, n_i (b_{1i}/\bar{b}_1)^a / \sum_{i=1}^{k} n_i (b_{1i}/\bar{b}_1)^a$$

where

n_i = number of survivors in initial dbh-class i(i=1,2, ,k)

n = total number of survivors ($n = \Sigma^k_{i=1} n_i$)

b_{1i} = basal area corresponding to midpoint of dbh-class i at age A_1

b_{2i} = basal area corresponding to midpoint dbh of the n_i survivors at age A_2

$\bar{b}_1$, $\bar{b}_2$ = average basal area of the n survivors at ages A_1 and A_2

$a = (A_2/A_1)^{0.05968}$

The stand table projection procedure requires a prior estimate of survival at age A_2, namely n, and of future per-acre basal area $B_2 = n\bar{b}_2$. In addition, the predicted total mortality must be identified in the initial stand table by dbh-class. This is accomplished by assuming that the probability that a tree in a given dbh-class will die during the projection interval is inversely proportional to its relative size defined as $(b_{1i}/\bar{b}_1)$. Having identified the trees predicted to die during the projection interval,as shown in Table 2, the relative sizes of the dbh-class midpoints are recalculated for the survivors only and are then projected according to equation 3.7 to obtain future dbh-class midpoints. A future stand table is obtained by assuming that survivors are uniformly distributed in each dbh-class and that class limits are halfway between successive dbh-class midpoints.

The procedure is illustrated for an example where an initial stand table is available for a 15-year-old plantation with an average dominant/codominant height of 48 ft, 495 trees per acre and a calculated basal area of 106 sq ft. per acre as shown in Table 1. The survival function of equation 2.3 predicts a survival of 400 at age 30 and the basal area projection equation 2.2 predicts 174 sq. ft. at age 30. The number of trees predicted to die in each dbh-class is calculated as shown in Table 2.

Table 2: Calculations to Predict Mortality by Dbh-Class

Dbh Class	Trees/ac 1	B/ac ft^2	$\bar{b}_1/b_{1i}$ 2	(2 ÷ Σ 2) 3	(1 ÷ Σ 1) 4	(3 x 4 ÷ Σ 3 x 4) 5	Mortality 5 x 95
2	1	0.02	9.83	.46	.002	.016	1
3	11	0.53	4.37	.20	.022	.083	8
4	43	3.79	2.46	.11	.087	.171	16
5	99	13.49	1.57	.07	.200	.252	24
6	143	28.02	1.09	.05	.289	.252	24
7	125	33.42	0.80	.04	.252	.162	15
8	59	20.68	0.61	.03	.119	.059	6
9	13	5.69	0.48	.02	.026	.010	1
10	1	0.55	0.39	.02	.002	.001	0
	495	106.19	21.61	1.00	1.0	1.0	95

$$\bar{b}_1 = \frac{106.19}{495} = 0.2145$$

Calculations to derive the projected stand table at age 30 with a projected survival of 400 and a projected basal area of 174 sq. ft. per acre are shown in Table 3. The projected stand and stock table are shown in Table 4.

Table 3: Calculations to Project the Stand Table

Dbh Class in.	Survivors* trees/ac n_i	B/ac ft^2	$n_i(b_{1i}/\bar{b}_1)^{1.0422}$** (1)	(1) ÷ Σ (1) (2)	$b_{2i} = 163\ (2)/n_i$ (3)	$d_{2i} = \sqrt{(3)/0.005454}$
3	3	0.15	0.63	.002	.0889	4.0
4	27	2.36	9.98	.025	.1619	5.4
5	75	10.23	44.59	.111	.2578	6.9
6	119	23.36	103.37	.258	.3769	8.3
7	110	29.40	131.28	.327	.5198	9.8
8	53	18.50	84.55	.211	.6866	11.2
9	12	5.30	24.34	.061	.8776	12.7
10	1	0.55	2.37	.006	1.0932	14.2
Total	400	89.84	401.11	1.0		

* Obtained from Table 2

$\bar{b} = 89.84/400 = 0.2246$

** $1.0422 = (A_2/A_1)^c = (30/15)^{0.05968}$

Table 4: Stand and Stock Table at age 30

Dbh Class in.	Trees/ac	B_i ft^2	H_i ft	V_i (2" top) ft^3
4	2*	0.17	44.7	3.1
5	15	2.05	53.9	40.4
6	30	5.89	61.3	136.5
7	52	13.90	67.1	354.3
8	80	27.93	71.7	752.7
9	79	34.90	75.4	996.5
10	75	40.91	78.3	1211.2
11	37	24.42	80.6	734.7
12	21	16.49	82.4	514.2
13	8	7.37	83.9	221.8
14	1	1.07	85.0	21.9
Total	400	175.1		4987

* The 3" dbh class midpoint is predicted to increase to 4.0" with class limits ± 0.7", that is, from 3.3" to 4.7". Of the 3 trees initially in the 3" dbh class

$$\frac{0.2}{1.4} \times 3 = .43 \text{ trees will remain in the 3" class}$$

$$\frac{1.0}{1.4} \times 3 = 2.14 \text{ trees will grow into the 4" class}$$

and

$$\frac{0.2}{1.4} \times 3 = 0.43 \text{ trees will grow into the 5" class}$$

3.2 Stand Table Projection for Thinned Plantations

In some instances a stand table may be available immediately before a thinning occurs. In such cases, the trees removed in the thinning can be identified in the stand table, and the remaining stand table projected to the desired future age. Suppose this was the case for the 15-year-old plantation with 495 surviving trees, an average dominant/codominant height of 48 ft. and a stand table as shown in Table 1 with a calculated basal area of 106 sq.ft. per acre. If this plantation were thinned selectively from below to 300 trees per acre, the basal area removed in thinning is predicted as in section 2.5 to be

$$B_t = 106\ (195/495)^{1.2248}$$

$$= 34 \text{ sq. ft. per acre}$$

For this method of thinning the expected number of trees removed from each dbh class can be calculated in a manner similar to mortality as shown in Table 2, by assigning selection probabilities inversely proportional to relative tree size, and adjusting the distribution if necessary to ensure that the total basal area removed equals 34 sq. ft. Calculations are illustrated in Table 5. A stand and stock table for the trees removed in thinning are shown in Table 6, and for the trees left after thinning in Table 7.

In the case of row thinnings the same proportion of trees are removed from each dbh class as the proportion of rows removed in the thinning operation. In the case of a combination row x selective thinning,

Table 5. Calculation of Trees Removed in Selective Thinning from Below

Dbh Class in.	Trees/ac	Prob* (1)	(1) x 195	Trees Thinned n_i	$n_i b_i$ (2)	(2) x 34/Σ (2)	d_i	Adj Trees Thinned	Trees left
2	1	.016	3	1	0.0218	0.0220	2.0	1	0
3	11	.083	16	11	0.5400	0.5459	3.0	11	0
4	43	.171	33	40	3.4906	3.5290	4.0	40	3
5	99	.252	49	49	6.6813	6.7547	5.0	49	50
6	143	.252	49	49	9.6211	9.7268	6.0	49	94
7	125	.162	32	32	8.5521	8.6461	7.0	32	93
8	59	.059	11	11	3.8400	3.8822	8.0	11	48
9	13	.010	2	2	0.8836	0.8933	9.0	2	11
10	1	.001	0	0	0			0	1
Total	495	1.0	195	195	33.6305	34.0000		195	300

* Probabilities calculated as shown in Table 2.

Table 6. Stand and Stock Table for Trees Removed

Dbh-Class in	Trees/ac	H_i ft.	B_i ft^2	V_i (2" Top) ft^3
2	1	19.2	0.02	0
3	11	29.3	0.55	5.0
4	40	36.5	3.53	45.9
5	49	41.8	6.75	104.2
6	49	45.5	9.73	165.4
7	32	48.2	8.65	156.5
8	11	50.2	3.88	73.3
9	2	51.6	0.89	17.4
Total	195		34.00	567.7

Table 7. Stand and Stock Table After Thinning

Dbh-Class in.	Trees/ac	H_i ft.	B_i ft^2	V_i (2" top) ft^3/ac
4	3	36.5	0.26	3.4
5	50	41.8	6.82	106.4
6	94	45.5	18.46	317.2
7	93	48.2	24.85	454.7
8	48	50.2	16.76	319.9
9	11	51.6	4.86	95.4
10	1	52.6	0.55	10.9
Total	300		72.56	1307.9

removal by row thinning is calculated first in the same manner as just described, and the selective removal is then calculated for the remaining stand table in the same manner as shown in Table 5.

In every case, the remaining stand table after thinning is projected to the desired future age as shown in Table 3. In this case with the stand table after thinning at age 15 to 300 trees per acre as shown in Table 7, the predicted survival at age 30 given by equation 2.3 is 250. The 50 trees predicted to die are identified as in Table 2, leading to the predicted mortality in each dbh class as shown in Table 8. Average dominant/codominant height at age 30 is obtained from equation 2.4 as 78', and per-acre basal area at age 30 from equation 2.2 as 128 sq. ft per acre. Calculations necessary to project the stand table to age 30 are shown in Table 3 and leads to the projected stand and stock tables in Table 8.

Table 8. Projected Stand and Stock Table at Age 30

Dbh Class in.	After Thin trees/ac	Mortality* trees/ac	Survivors trees/ac	Projected** trees/ac	H_i ft	B_i/ac ft^2/ac	V_i (2" top) ft^3/ac
4	3	1	2				
5	50	13	37	1	50.7	0.11	2.1
6	94	17	77	4	58.1	0.83	18.2
7	93	13	80	25	64.2	6.61	160.1
8	48	5	43	42	69.0	14.54	379.7
9	11	1	10	52	73.0	22.77	628.8
10	1		1	54	76.2	29.27	843.6
11				37	78.8	24.63	733.6
12				26	80.9	20.42	634.3
13				7	82.5	6.13	191.1
14				3	83.9	3.13	99.2
Total	300	50	250	250		128.44	3690.7

* Calculated as in Table 2.

** Calculated as in Table 3.