Merchantable-stem Green and Dry Weight Prediction Equations Based on a New Segmented-stem Taper Model

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[1] This system of weight prediction models is a follow-up work to the segmented-stem taper and volume system reported by Fang, Borders and Bailey in PMRC Technical Report 1999-3.

Executive Summary

Based on taper measurements and green and dry weight data for plantation-grown slash (*Pinus elliottii* Engelm.) and loblolly pine (*Pinus taeda* L.), a new system of simultaneous taper, volume and weight models produces improved fitting statistics when used to predict tree volume and weight. The taper-volume models from a previous work (Fang, Borders and Bailey 2000) combined with an equation for weight per unit volume provide compatible predictions of green and dry weight to variable top diameters. The predictions for green and dry weight vary in a logical relationship with age of the tree and distance above the ground. Substantial improvements in statistics of fit over those for prior PMRC models are realized with this approach to weight equations. The old models (Harrison and Borders 1989) for merchantable-stem dry weight fitted with these data produced adjusted R^2 values of .87 and .88 for loblolly and slash, respectively. The dry weight prediction model in this system produced adjusted R^2 values of .98 and .99 for loblolly and slash.

Introduction

Attempts to model the form of an individual tree date from the early 20th century (Behre 1923, 1927). They are characterized by two approaches: (a) express variable form as a single continuous function or (b) expresses variable form with a step function in such a way that the bole is separated into segments by inflection points with form being constant within each segment and different between segments. Examples of both approaches abound in the literature. Typically, a bit of modeling logic leads to an equation form that is fitted to taper measurements. Subsequently, integral calculus may be applied to derive a volume equation from the taper model. In a utilitarian departure from models for stem taper, Burkhart (1977) introduced the concept of an equation to predict volume to any merchantable top limit with top limit diameter itself entering as a predictor variable in the model along with diameter at breast height (dbh) and total height. His equation allows one to predict total volume as a special case of merchantable volume when top diameter equals zero. Following this work, Clutter (1980) showed that such models as Burkhart's (1977) may be analyzed using the fundamental theorem of calculus to develop an implied taper equation.

Recently Fang *et al.* (2000) presented a new example of the segmented-bole type of taper model that gave excellent results for plantation-grown slash (*Pinus elliottii* Engelm.) and loblolly pine (*Pinus taeda* L.). The volume equation developed in this work also has the ability to predict the volume to any upper-stem diameter or merchantable height for a given dbh and total tree height with total volume as a special case.

The advent of weight scaling (Taras 1956) and the realization of its many advantages stimulated interests in tree measurational models for predicting green and dry weight to various merchantable top diameter limits. Most such equations directly predict the weight to some merchantable top diameter for given inputs of dbh, total height, and the merchantable top diameter much the same as the variable-top volume equation does. For example, Harrison and Borders (1996) use a similar form of equation for weight prediction as their model for variable-top volume predictions, which is itself similar to the variable-top volume model introduced by Burkhart (1977).

Parresol and Thomas (1989) introduced an altogether different approach to weight prediction based on the concept of the mass of a lamina with a continuous, constant density. Working with data from slash pine plantations, they provide a linear regression equation to predict specific gravity as a function of relative height (i.e. merchantable height divided by total height) and tree age. They also fitted the taper model of Kozak et al. (1969) with data from plantation-grown slash pine trees. By integrating the product of lamina times specific gravity over the limits in relative height, Parresol and Thomas (1989) derive a merchantable-stem weight prediction equation.

Objective

An innovative system for taper-volume models based on segmented-stems with different form factors for each segment (Fang *et al*. 2000) has given rise to new thinking regarding volume-taper relationships. However, as noted above, the timber industry has moved away from volumetric measure in favor of weight. Thus, our objective in this work was to extend the system of Fang *et al.* (2000) to include variable-top weight prediction as a compatible and logical extension of the system. We were particularly interested in the density-integral approach of Parresol and Thomas (1989) and the variable-top weight prediction equation presented by Harrison and Borders (1996). We also wanted to explore other alternatives, such as directly predicting the average weight per ft^3 for variable stem segments.

The data

For this work we used weight data from the same loblolly (*Pinus taeda* L.) and slash pine (*Pinus elliottii* Engelm.) data sets described by Fang *et al.* (2000). Data were available from 1280 individual loblolly pine trees obtained from 376 sample plots located in plantations in the coastal plain and piedmont physiographic provinces of North Carolina, South Carolina, Georgia, Florida and Alabama. Measurements were available on 871 individual slash pine trees from 256 plantation plots in the coastal plain of Georgia and north Florida. In most cases, four sample trees without any obvious stem abnormalities were felled on each sample plot.

Tree selection and measurement protocols are described by Fang *et al.* (2000). The stem-analysis measurements taken at 5-ft intervals on the stems were used in the overlapping-bolts method (Bailey 1995) to determine observed volume to variable top diameters. From a disk taken at the stump and the top of each bolt, green and dry weights in lbs per ft^3 were determined. After these calculations, there were a total of 11,359 observations at variable top limits for the taper-volume-weight system for loblolly pine and 8,345 observations for the slash pine.

For the loblolly pine trees, age averaged 14 years and ranged from 9 to 26; dbh averaged 6.5 in. and ranged from 1.3 to 13.6 in.; and total tree height averaged 43 ft and ranged from 13 to 85 ft. For the slash pine sample trees, age averaged 15 years and ranged from 9 to 27; dbh averaged 6.3 in. and range from 2.6 to 13.5 in.; and total tree height averaged 45 ft and ranged from 19 to 76 ft.

The Taper-volume Models

The taper-volume model system of Fang *et al.* (2000) can be summarized as follows:

$$d = c_1 \sqrt{H^{(k-b_1)/b_1} (1-z)^{(k-b)/b} a_1^{I_1+I_2} a_2^{I_2}} \qquad (1)$$

$$V_m = c_1^2 H^{k/b_1} [b_1 t_0 + (I_1 + I_2)(b_2 - b_1) t_1 + I_2 (b_3 - b_2) a_1 t_2 - b (1 - z)^{k/b} a_1^{I_1+I_2} a_2^{I_2}] \qquad (2)$$

where

$$c_1 = \sqrt{a_0 D^{a_1} H^{a_2 - k/b_1} / [b_1 (t_0 - t_1) + b_2 (t_1 - a_1 t_2) + b_3 a_1 t_2]} \quad ,$$

$$t_0 = (1 - p_0)^{k/b_1} ,$$

$p_0 = h_0/H$, the stump height ratio;

$$t_1 = (1 - p_1)^{k/b_1} \quad ,$$

$$t_2 = (1 - p_2)^{k/b_2} \quad ,$$

$$a_1 = (1 - p_1)^{(b_2 - b_1)k/b_1 b_2} \quad ,$$

$$a_2 = (1 - p_2)^{(b_3 - b_2)k/b_2 b_3} \quad ,$$

$$b = b_1^{1-(I_1+I_2)} b_2^{I_1} b_3^{I_2} \quad ,$$

$$I_1 = \begin{cases} 1 & \textit{If} \quad p_1 < z \le p_2 \\ 0 & \textit{Otherwise} \end{cases} \quad ,$$

$$I_2 = \begin{cases} 1 & \textit{If} \quad p_2 < z \le 1 \\ 0 & \textit{Otherwise} \end{cases} \quad ,$$

$z = h / H$, and

k = 0.005454154 is the English constant to convert $in^2\,ft$ into ft^3.

D = diameter at breast height (in.),

H = total tree height (ft),

d = upper-stem diameter (in.) at height h,

h = the length (ft) from ground to upper-stem diameter d, and

h_1 and h_2, are the length (ft) from ground to inflection point 1 and 2, p_1 and p_2 are corresponding height ratio. Estimates for the coefficients in this system are, of course, provided by Fang *et al*. (2000). We will not present those coefficients here because we have refitted the system as a simultaneous set of equations including a weight model component.

We compared the fitting statistics of our new taper-volume-weight system to those arising from fitting the taper-weight modeling approach of Parresol and Thomas (1989) and the variable-top weight prediction equation of Harrison and Borders (1996).

Weight Model Components

Similarly to the specific gravity prediction equation developed by Parresol and Thomas (1989), we hypothesized the following model:

$$\rho = a + bh + cA \quad , \qquad (3)$$

where ρ is the average lbs/ft^3 for the stem segment from stump height to a distance *h* (ft) from the ground, *A* is the tree age in years, and (*a, b, c*) are regression coefficients to be estimated with the data. Of course, the regression coefficient estimates will be unique depending on the prediction of green or dry weight. Once these coefficient estimates are known, the weight prediction equation arises from the integral of the taper model and equation (3):

$WT = \int_{h_0}^{h} k\rho\, d^2 dh$, where *WT*= weight of tree stem include between the heights h_0 and h. Substitution of the appropriate equations from above produces

$$\begin{aligned}
\int k\rho\, d^2 dh &= \int k(a + bh + cA)c_i^2 (H-h)^{(k-\beta_i)/\beta_i} dh \\
&= \int k c_i^2 (a + cA)(H-h)^{(k-\beta_i)/\beta_i} dh + \int k c_i^2 bh (H-h)^{(k-\beta_i)/\beta_i} dh \\
&= -c_i^2 \beta_i (a + cA)(H-h)^{k/\beta_i} - c_i^2 \beta_i bh (H-h)^{k/\beta_i} - c_i^2 \beta_i \frac{b\beta_i^2}{k+\beta_i}(H-h)^{k/\beta_i+1} \\
&= -c_i^2 \beta_i (H-h)^{k/\beta_i}\left(a + cA + bh + \frac{b\beta_i}{k+\beta_i}(H-h)\right)
\end{aligned}$$

Since there are three segments for the taper equation, the merchantable weight is obtained for three distinct segments:

Segment 1: if $p_0H \le h < p_1H$,

$$WT = \int_{p_0H}^{h} k\rho\, d^2 dh = -c_1^2 \beta_1 (H-h)^{k/\beta_1}\left(a + cA + bh + \frac{b\beta_1}{k+\beta_1}(H-h)\right)\Bigg|_{h=p_0H}^{h=h}$$

$$= c_1^2 H^{k/\beta_1}\left(\beta_1(1-p_0)^{k/\beta_1}[a + cA + bp_0H + \frac{b\beta_1 H}{k+\beta_1}(1-p_0)] - \beta_1(1-z)^{k/\beta_1}[\rho + \frac{b\beta_1 H}{k+\beta_1}(1-z)]\right)$$

Segment 2: if $p_1H \le h < p_2H$,

$$WT=\int_{p_0H}^{p_1H}kr\,d^2dh+\int_{p_1H}^{h}kr\,d^2dh=-c_1^2b_1(H-h)^{k/b_1}\left(a+cA+bh+\frac{bb_1}{k+b_1}(H-h)\right)\Bigg|_{h=p_0H}^{h=p_1H}$$

$$-c_2^2b_2(H-h)^{k/b_2}\left(a+cA+bh+\frac{bb_2}{k+b_2}(H-h)\right)\Bigg|_{h=p_1H}^{h=h}$$

$$=c_1^2H^{k/b_1}\left(\begin{array}{l}b_1(1-p_0)^{k/b_1}[a+cA+b\,p_0H+\frac{bb_1H}{k+b_1}(1-p_0)]-b_1(1-p_1)^{k/b_1}[a+cA+b\,p_1H+\frac{bb_1H}{k+b_1}(1-p_1)]\\+b_2(1-p_1)^{k/b_1}[a+cA+b\,p_1H+\frac{bb_2H}{k+b_2}(1-p_1)]-b_2a_1^2(1-z)^{k/b_2}[r+\frac{bb_2H}{k+b_2}(1-z)]\end{array}\right)$$

$$=c_1^2H^{k/b_1}\left(\begin{array}{l}b_1(1-p_0)^{k/b_1}[a+cA+b\,p_0H+\frac{bb_1H}{k+b_1}(1-p_0)]+(1-p_1)^{k/b_1}(a+cA+b\,p_1H)(b_2-b_1)+\\(1-p_1)^{k/b_1+1}bH(\frac{b_2^2}{k+b_2}-\frac{b_1^2}{k+b_1})-a_1^2\,b_2(1-z)^{k/b_2}[r+\frac{bb_2H}{k+b_2}(1-z)]\end{array}\right)$$

Segment 3: if $p_2H \le h \le H$

$$WT=\int_{p_0H}^{p_1H}kr\,d^2dh+\int_{p_1H}^{p_2H}kr\,d^2dh+\int_{p_2H}^{h}kr\,d^2dh$$

$$=-c_1^2b_1(H-h)^{k/b_1}\left(a+cA+bh+\frac{bb_1}{k+b_1}(H-h)\right)\Bigg|_{h=p_0H}^{h=p_1H}$$

$$-c_2^2b_2(H-h)^{k/b_2}\left(a+cA+bh+\frac{bb_2}{k+b_2}(H-h)\right)\Bigg|_{h=p_1H}^{h=p_2H}$$

$$-c_3^2b_3(H-h)^{k/b_3}\left(a+cA+bh+\frac{bb_3}{k+b_3}(H-h)\right)\Bigg|_{h=p_2H}^{h=h}$$

$$=c_1^2H^{k/b_1}\left(\begin{array}{l}b_1(1-p_0)^{k/b_1}[a+cA+b\,p_0H+\frac{bb_1H}{k+b_1}(1-p_0)]+(1-p_1)^{k/b_1}(a+cA+b\,p_1H)(b_2-b_1)+\\(1-p_1)^{k/b_1+1}bH(\frac{b_2^2}{k+b_2}-\frac{b_1^2}{k+b_1})+a_1^2(1-p_2)^{k/b_2}(a+cA+b\,p_2H)(b_3-b_2)+\\a_1^2(1-p_2)^{k/b_2+1}bH(\frac{b_3^2}{k+b_3}-\frac{b_2^2}{k+b_2})-a_1^2a_2^2\,b_3(1-z)^{k/b_3}[r+\frac{bb_3H}{k+b_3}(1-z)]\end{array}\right)$$

Let:

$$q_0=b_1(1-p_0)^{k/b_1}[a+cA+b\,p_0H+\frac{bb_1H}{k+b_1}(1-p_0)],$$

$$q_{12}=(1-p_1)^{k/b_1}(a+cA+b\,p_1H)(b_2-b_1)$$

$$q_{23} = \alpha_1^{2}(1 - p_2)^{k/b_2}\,(a + c\,A + b\,p_2 H)(b_3 - b_2)\,,$$

$$s_{12} = (1 - p_1)^{k/b_1 + 1} bH\left(\frac{b_2^2}{k + b_2} - \frac{b_1^2}{k + b_1}\right)$$

$$s_{23} = \alpha_1^{2}(1 - p_2)^{k/b_2 + 1} bH\left(\frac{b_3^2}{k + b_3} - \frac{b_2^2}{k + b_2}\right)$$

$$s_1 = \frac{bb_1 H}{k + b_1}\,,\quad s_2 = \frac{bb_2 H}{k + b_2}\,,\quad s_3 = \frac{bb_3 H}{k + b_3}\,,$$

and with these and previously defined terms, we can write the merchantable weight equation in a compact form:

$$WT = c_1^2 H^{k/b_1}\begin{pmatrix}(1 - I_1 - I_2)\left(q_0 - b(1 - z)^{k/b}[r + s_1(1 - z)]\right) \\ + I_1\left(q_0 + q_{12} + s_{12} - b\alpha_1^2(1 - z)^{k/b}[r + s_2(1 - z)]\right) \\ + I_2\left(q_0 + q_{12} + s_{12} + q_{23} + s_{23} - b\alpha_1^2\alpha_2^2(1 - z)^{k/b}[r + s_3(1 - z)]\right)\end{pmatrix} \qquad (4)$$

Since the weight equation is obtained by integrating the compatible taper equation, all the properties of compatibility in the taper-volume system are preserved. Moreover, the resulting weight prediction equation inherits these nice properties as well.

Finally, the compatible taper-volume–weight model system based on a three-segment stem taper model can be summarized as follows:

$$d = c_1\sqrt{H^{(k - b_1)/b_1}(1 - z)^{(k - b)/b}\,\alpha_1^{I_1 + I_2}\alpha_2^{I_2}}$$

$$V_m = c_1^2 H^{k/b_1}[b_1 t_0 + (I_1 + I_2)(b_2 - b_1)t_1 + I_2(b_3 - b_2)\alpha_1 t_2 - b(1 - z)^{k/b}\alpha_1^{I_1 + I_2}\alpha_2^{I_2}]$$

$$WT = c_1^2 H^{k/b_1}\begin{pmatrix}(1 - I_1 - I_2)\left(q_0 - b(1 - z)^{k/b}[r + s_1(1 - z)]\right) \\ + I_1\left(q_0 + q_{12} + s_{12} - b\alpha_1^2(1 - z)^{k/b}[r + s_2(1 - z)]\right) \\ + I_2\left(q_0 + q_{12} + s_{12} + q_{23} + s_{23} - b\alpha_1^2\alpha_2^2(1 - z)^{k/b}[r + s_3(1 - z)]\right)\end{pmatrix}$$

$$h = H\left(1 - \left[\frac{d^2}{c_1^2}H^{(b_1 - k)/b_1}\alpha_1^{-(I_1 + I_2)}\alpha_2^{-I_2}\right]^{b/(k - b)}\right)$$

$$V = a_0 D^{a_1} H^{a_2}$$

$$W = q_0 + q_{12} + s_{12} + q_{23} + s_{23}$$

Where: D = diameter at breast height (in.),
H = total tree height (ft.),
A = tree age (yrs),
d = upper-stem diameter (in.) at height h (ft.),
h = the length (ft.) from ground to upper-stem diameter d, and
V = total tree volume (ft^3)

W = total tree weight (lb.)
V_m = Merchantable volume (ft^3) to an upper-stem diameter of d
W_m = Merchantable weight (lb.) to an upper-stem diameter of d
k = the English constant, 0.005454154

$$r = a + bh + cA$$

$$c_1 = \sqrt{a_0 D^{a_1} H^{a_2 - k/b_1} / [b_1(t_0 - t_1) + b_2(t_1 - \alpha_1 t_2) + b_3 \alpha_1 t_2]} \quad ,$$

$p_0 = h_0/H$, $z = h/H$,

$$t_0 = (1 - p_0)^{k/b_1},$$

$$t_1 = (1 - p_1)^{k/b_1} \quad ,$$

$$t_2 = (1 - p_2)^{k/b_2} \quad ,$$

$$\alpha_1 = (1 - p_1)^{(b_2 - b_1)k/b_1 b_2} \quad ,$$

$$\alpha_2 = (1 - p_2)^{(b_3 - b_2)k/b_2 b_3} \quad ,$$

$$\beta = b_1^{1-(I_1+I_2)} b_2^{I_1} b_3^{I_2} \quad ,$$

$$q_0 = b_1(1 - p_0)^{k/b_1}[a + cA + bp_0H + \frac{bb_1H}{k + b_1}(1 - p_0)],$$

$$q_{12} = (a + cA + bp_1H)(b_2 - b_1)$$

$$q_{23} = (a + cA + bp_2H)(b_3 - b_2),$$

$$s_{12} = (1 - p_1)^{k/b_1+1} bH(\frac{b_2^2}{k + b_2} - \frac{b_1^2}{k + b_1})$$

$$s_{23} = \alpha_1^2 (1 - p_2)^{k/b_2+1} bH(\frac{b_3^2}{k + b_3} - \frac{b_2^2}{k + b_2})$$

$$s_1 = \frac{bb_1H}{k + b_1},$$

$$s_2 = \frac{bb_2H}{k + b_2},$$

$$s_3 = \frac{bb_3H}{k + b_3},$$

If bolt height is given then:

$$I_1 = \begin{cases} 1 & \text{If } p_1 < z \le p_2 \\ 0 & \text{Otherwise} \end{cases} \quad , \qquad I_2 = \begin{cases} 1 & \text{If } p_2 < z \le 1 \\ 0 & \text{Otherwise} \end{cases}$$

If upper diameter limit is given then:

$$I_1 = \begin{cases} 1 & \text{If } D_2 \le d < D_1 \\ 0 & \text{Otherwise} \end{cases} \quad , \qquad I_2 = \begin{cases} 1 & \text{If } d < D_2 \\ 0 & \text{Otherwise} \end{cases}$$

$$D_1 = c_1\sqrt{[H(1 - p_1)]^{(k-b_1)/b_1}} \quad , \quad D_2 = c_1\sqrt{\alpha_1 H^{(k-b_1)/b_1}(1 - p_2)^{(k-b_2)/b_2}}$$

Parameter Estimation

With each of the two sets of data, equations (1), (2), and (4) were fitted as a simultaneous system (Borders 1989) using full-information maximum likelihood with initial parameter estimates provided by OLS and SUR. Initial results with unweighted fitting suggested residuals should be weighted using upper-stem heights of the individual bolts. In order to have the weights sum to the number of observations, the weights used were

$$w_i = k/(\text{bolt height}),$$

where the constant k is determined for each of the six data set and volume-weight combinations so that $\sum_{i=1}^{\#observations} w_i$ equals the sum of observations.

For the resulting six equations, all produced remarkably good statistics of fit with the lowest adjusted R^2 value being .97 for the loblolly inside-bark taper equation (Tables 1 -- 4). Plots of weighted residuals over bolt height showed no trends and uniformity of variance across height values. No relationships between weighted residuals and other variables were apparent. In addition, the estimates for parameter *b* in equation (3) show logical trends. Predicted dry wood density decreases ($b < 0$) as height up the tree (*h*) increases indicating the effect of an increased percentage of juvenile wood whereas green wood density increases ($0 < b$) as height increases indicating the effect of an increase in moisture content. The relationship on tree age indicates an increase ($0 < c$) in dry weight density with age indicative of a higher proportion of late wood and, hence, a higher specific gravity. Green weight density is predicted to decrease with age. This would indicate that the effect of a higher moisture content for the younger tree offsets the effect of a higher specific gravity of wood in the older tree. Although the net effect is small ($c = -0.09$ lb/ft^3/yr for loblolly and $c = -0.07$ lb/ft^3/yr for slash), it is not illogical.

Model Comparisons

Summary statistics (Tables 5 and 6) from fitting the taper-weight model of Parresol and Thomas (1989) and the variable-top weight prediction equation of Harrison and Borders (1996) indicate very close agreement in precision between our system and that of Parresol and Thomas. However, just as did Parresol and Thomas, we found the lamina integral approach to developing a weight equation superior to the variable-top weight prediction model. In our case, we used the Harrison and Borders equation for comparison. It is similar in structure to the “weight ratio” approach Parresol and Thomas compared their system with.

Summary

Extending the segmented-stem form factors system of Fang, Borders and Bailey (2000) to include a provision for predicting weight is a logical next step. By starting with a relationship between density (green or dry) and a linear function of stem height and tree age, the lamina integral approach introduced by Parresol and Thomas (1989) results in equations for green weight (outside bark) and dry weight (inside bark). The system fits very well with data from site-prepared loblolly and slash pine plantations with results that are superior to those from fitting a variable-top weight equation.

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Table 1. SAS System statistics of fit for loblolly pine site-prepared plantations: inside-bark diameter (dib, in.), inside-bark merchantable volume (MCVIB, ft^3), and inside-bark dry weight (MCDRYWT, lbs).

Nonlinear FIML Summary of Residual Errors

Equation	DF Model	DF Error	SSE	MSE	Root MSE	R-Square	Adj R-Sq
dib	2.667	11415	2237.0	0.1960	0.4427	0.9720	0.9720
MCVIB	2.667	11415	315.2	0.0276	0.1662	0.9913	0.9913
MCDRYWT	5.667	11412	551792	48.3505	6.9534	0.9820	0.9820

Parameter	Estimate	Approx Std Err	t Value	Approx Pr > |t|
a0	0.001333	9.122E-6	146.15	<.0001
a1	2.01886	0.00219	919.81	<.0001
a2	1.088438	0.00242	450.43	<.0001
b1	0.000832	9.711E-6	85.66	<.0001
b2	0.00268	0.000019	140.47	<.0001
b3	0.001941	0.000045	42.69	<.0001
p1	0.083535	0.00182	45.98	<.0001
p2	0.567321	0.0151	37.47	<.0001
a	24.8262	0.0460	539.24	<.0001
b	-0.03768	0.000943	-39.96	<.0001
c	0.267727	0.00200	133.61	<.0001

Number of Observations 11,418

Table 2. SAS System statistics of fit for slash pine site-prepared plantations: inside-bark diameter (dib, in.), inside-bark merchantable volume (MCVIB, ft^3), and inside-bark dry weight (MCDRYWT, lbs).

Nonlinear FIML Summary of Residual Errors

Equation	DF Model	DF Error	SSE	MSE	Root MSE	R-Square	Adj R-Sq
dib	2.667	8342	1451.1	0.1739	0.4171	0.9732	0.9732
MCVIB	2.667	8342	184.9	0.0222	0.1489	0.9924	0.9924
MCDRYWT	5.667	8339	243171	29.1596	5.4000	0.9905	0.9905

Parameter	Estimate	Approx Std Err	t Value	Approx Pr > |t|
a0	0.001895	0.000020	95.82	<.0001
a1	2.04954	0.00305	671.56	<.0001
a2	0.987433	0.00390	253.16	<.0001
b1	0.000796	0.000013	62.25	<.0001
b2	0.002799	0.000029	98.16	<.0001
b3	0.002181	0.000044	49.99	<.0001
p1	0.076645	0.00209	36.63	<.0001
p2	0.520421	0.0220	23.69	<.0001
a	29.96699	0.1061	282.33	<.0001
b	-0.04046	0.00125	-32.33	<.0001
c	0.166596	0.00521	31.97	<.0001

Number of Observations 8,344

Table 3. SAS System statistics of fit for loblolly pine site-prepared plantations: outside-bark diameter (dob, in.), ourside-bark merchantable volume (MCVOB, ft^3), and outside-bark green weight (MCGRWT, lbs).

Nonlinear FIML Summary of Residual Errors

Equation	DF Model	DF Error	SSE	MSE	Root MSE	R-Square	Adj R-Sq
dob	2.667	11356	1862.0	0.1640	0.4049	0.9804	0.9804
MCVOB	2.667	11356	298.9	0.0263	0.1622	0.9943	0.9943
MCGRWT	5.667	11353	1998843	176.1	13.2687	0.9877	0.9877

Parameter	Estimate	Approx Std Err	t Value	Approx Pr > |t|
a0	0.003597	0.000019	186.11	<.0001
a1	1.926405	0.00176	1091.97	<.0001
a2	0.941025	0.00195	482.08	<.0001
b1	0.000803	8.172E-6	98.27	<.0001
b2	0.002478	0.000013	189.43	<.0001
b3	0.00193	0.000052	37.21	<.0001
p1	0.091155	0.00172	53.13	<.0001
p2	0.610376	0.0182	33.58	<.0001
a	56.238	0.1610	349.20	<.0001
b	0.058952	0.00187	31.45	<.0001
c	-0.09241	0.00924	-10.00	<.0001

Number of Observations 11,360

Table 4. SAS System statistics of fit for slash pine site-prepared plantations: outside-bark diameter (dob, in.), ourside-bark merchantable volume (MCVOB, ft^3), and outside-bark green weight (MCGRWT, lbs).

Nonlinear FIML Summary of Residual Errors

Equation	DF Model	DF Error	SSE	MSE	Root MSE	R-Square	Adj R-Sq
dob	2.667	8342	1328.1	0.1592	0.3990	0.9806	0.9806
MCVOB	2.667	8342	185.9	0.0223	0.1493	0.9952	0.9952
MCGRWT	5.667	8339	697620	83.6541	9.1463	0.9943	0.9943

Parameter	Estimate	Approx Std Err	t Value	Approx Pr > |t|
a0	0.004444	0.000036	124.72	<.0001
a1	1.98821	0.00229	866.47	<.0001
a2	0.871207	0.00299	291.13	<.0001
b1	0.000628	7.069E-6	88.88	<.0001
b2	0.002704	0.000019	145.82	<.0001
b3	0.002144	0.000045	47.46	<.0001
p1	0.072003	0.00122	58.81	<.0001
p2	0.573215	0.0202	28.44	<.0001
a	56.92814	0.1448	393.23	<.0001
b	0.032636	0.00162	20.10	<.0001
c	-0.07253	0.00737	-9.84	<.0001

Number of Observations 8,344

Table 5. – A comparison of fit statistics for three modeling systems in their ability to predict inside bark dry weight (ft^3) of loblolly pine in site-prepared plantations. All three models fitted with full information maximum likelihood using SAS. Residuals were weighted by bolt height.

Model	# parm	MSE	root MSE (lb.)	*Mean* Res. (lb.)	Std Res. (lb.)	R^2	Adj. R^2
Segmented-stem model	**5.67***	**48.4**	**6.953**	**0.261**	**6.947**	**0.9820**	**0.9820**
Harrison & Borders (1996)	**5**	**343**	**18.53**	**2.811**	**18.31**	**0.8723**	**0.8723**
Parresol & Thomas (1989)	**6**	**48.5**	**6.965**	**-0.121**	**6.963**	**0.9820**	**0.9819**

*** Since 8 parameters a_0, a_1, a_2, b_1, b_2, b_3, p_1 and p_2 are shared by the 3 components of taper, merchantable volume and merchantable weight, in simultaneous estimation the number of parameters for those counts as 8/3=2.667 in each component in the system. So for the weight equation, the total number of parameters is 3 (a,b,c in r) + 2.667 = 5.667.**

Table 6 – A comparison of fit statistics for three modeling systems in their ability to predict inside bark dry weight (ft^3) of slash pine in site-prepared plantations. All three models fitted with full information maximum likelihood using SAS. Residuals were weighted by bolt height.

Model	# parm	MSE	root MSE (lb.)	*Mean* Res. (lb.)	Std Res. (lb.)	R^2	Adj. R^2
Segmented-stem model	**5.67***	**29.15**	**5.400**	**0.475**	**5.377**	**0.9905**	**0.9905**
Harrison & Borders (1996)	**5**	**358**	**18.92**	**2.915**	**18.69**	**0.8829**	**0.8828**
Parresol & Thomas (1989)	**6**	**28.13**	**5.304**	**0.311**	**5.294**	**0.9908**	**0.9908**

*** Since 8 parameters a_0, a_1, a_2, b_1, b_2, b_3, p_1 and p_2 are shared by the 3 components of taper, merchantable volume and merchantable weight, in simultaneous estimation the number of parameters for those counts as 8/3=2.667 in each component in the system. So for the weight equation, the total number of parameters is 3 (a,b,c in r) + 2.667 = 5.667.**