SOLUTIONS FOR REDUCING BIAS IN INVENTORY PROJECTIONS WHEN USING A SYSTEM OF INCOMPATIBLE SITE INDEX AND HEIGHT MODELS: DEMONSTRATED EXAMPLE USES JACK PINE OF ONTARIO

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Abstract

A fixed base age site index is commonly used as a covariate in height models. Site index models have been developed to predict the site index when it is unknown. When the site index and height equations are incompatible, a large bias may result in the height prediction when using the site index equation to predict the site index for use in the height equation. This problem is illustrated using as an example recently published models with incompatible site index and height equations. Furthermore, it is demonstrated using several methods that the bias is reduced. First, assuming that the primary objective is to predict height, the site index equation parameters, using pseudo-data generated from the height equation, are re-estimated by minimizing errors in height predictions while holding the published height model parameters constant. This reduces the bias and the degree of incompatibility between the site index and height equations. Second, dynamic equations are evaluated as an alternative to the fixed base-age concept. An existing lodgepole pine dynamic equation substantially reduced the bias in inventory projection. In addition, a dynamic equation was fitted to the pseudo-data and substantially reduced the bias for inventory projections relative to the other models. The dynamic models produced less biased, more parsimonious, and flexible solutions.

Introduction

Site dependent height growth models and their self-referencing functions are an integral part of forest growth and yield predictions. Fixed base-age site index is commonly used as a covariate in the height growth model. When the height model can not be inverted, i.e., a closed form solution for site index as a function of height can't be obtained, then generally one of the following techniques is used to estimate site index. One approach is to use an iterative algorithm to solve for site index given the height and age (e.g., Biging and Wensel 1985, Shaw and Packee 1998). This approach is based on the assumption that given a height and age, there is a site index that corresponds to the height and age and passes through the curve. Another approach is to develop a separate site index model by fitting the data using site index as a function of height and age (Carmean et al. In Press, Monserud 1984). This approach is based on Curtis et al. (1974), which advocated that the most efficient estimates of height from site index and age, and site index from height and age are from separate models. There have been numerous approaches to modeling height growth and they can be generally classified as static or dynamic models. Static site index height models are the most widely used for modeling height growth in forestry. Incompatible site index and height models may result when using static equations. In contrast, dynamic models are relatively scarce for modeling height in forestry, e.g., Bailey and Clutter 1974, Cieszewski and Bella, 1989, Cieszewski, 2001. The dynamic equations can be described as initial condition difference equations that are capable of using variable base ages. When based on proper algebra, these models are base age invariant (Bailey and Clutter, 1974). Thus selection of an arbitrary base age has no effect on the height predictions.

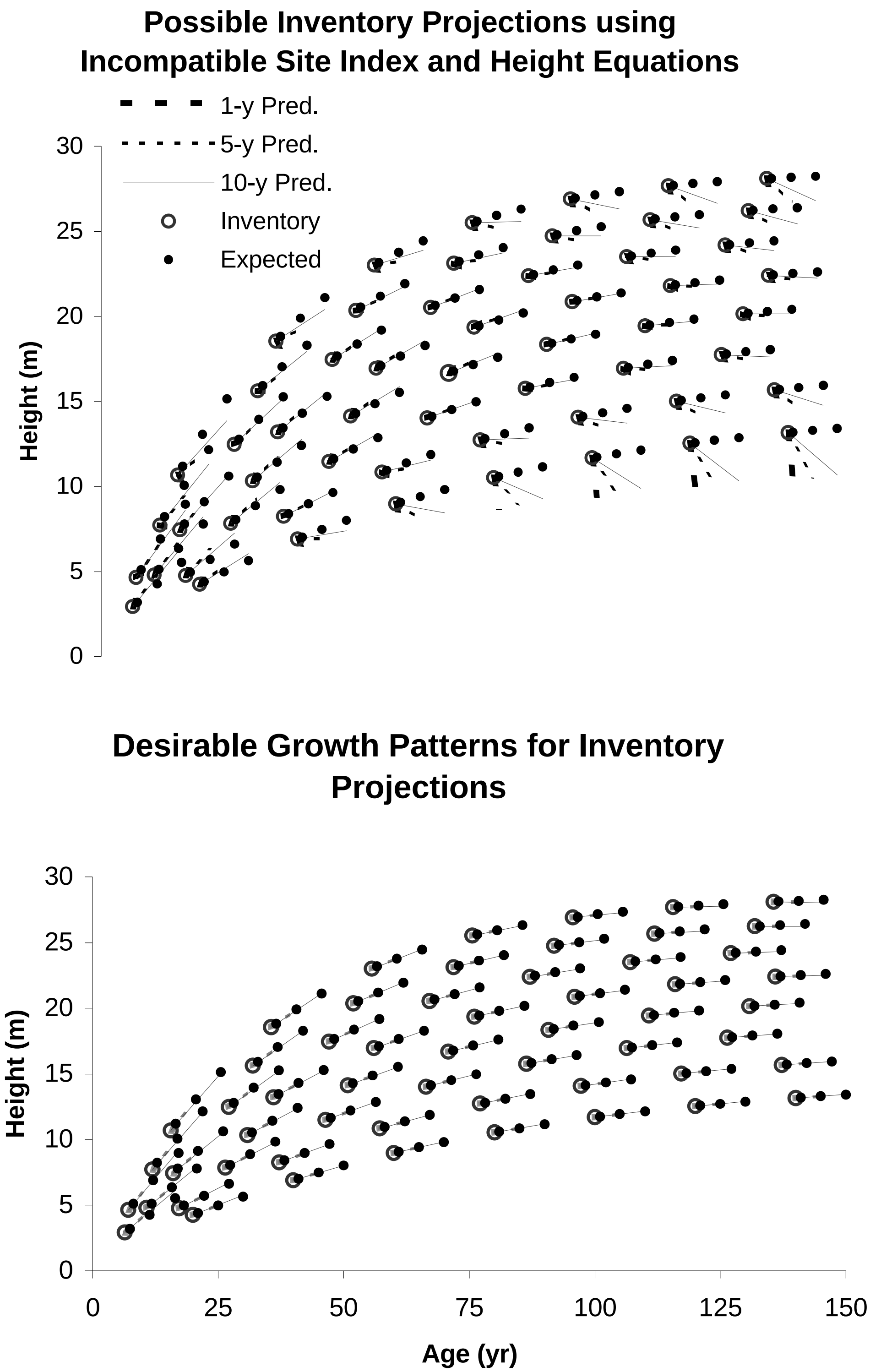


Figure 1. An illustration of the possible consequences when using an incompatible system of site index and height equations and desirable one, five, and ten-year inventory projections.

The ability to accurately project the inventory is a crucial component in the forestry decision process. The implementation of a forestry management regime depends upon accurate forest inventory projections. The height prediction model is a necessary component for any forest growth and yield system. Regardless of the management objectives for a particular forest, the ability to accurately assess the future forest structure is dependent upon the ability to predict height, i.e. inventory projections. The top graph in Figure 1 illustrates the possible consequences of a random inventory projection when using a system of incompatible site index and height equations, i.e., it is conceivable that projected heights could decrease from the measured height. The lower graph illustrates the desired type of inventory projections, i.e., the heights are increasing for each projection interval. The objective of this study is to develop methods for improving site index predictions given a system of incompatible site index and height models so that the inventory projected heights increase relative to the measured heights. The improved equations will result in a bias reduction for the height predictions given the height-age relationship.

Assessment of Recent Jack Pine Site Index and Height Growth Models for Ontario

Carmean et al. (In Press) present site index and height growth models for jack pine of Ontario that consists of two independent equations. These equations were developed separately for height and site index following Curtis et al. (1974). The models were developed from jack pine stands of Northern Ontario using stem analysis data. The data for developing the height and site index models came from 383 fully-stocked, even-aged, undisturbed jack pine plots. Their study defined site index as the average height of the dominant and codominant trees 50 years after they reached breast height (1.3 m). The observed jack pine site indices for their study ranged from 7.6 m to 22.4 m. The oldest plots were 157 years and the jack pine stands of this region tend to deteriorate at about age 70-80. They included 42 stands older than 100 years, but most plots were age 50-80. The jack pine distribution by site index class from the Carmean et al. (In Press) study reveals that the weighted average (approximate) using the midpoints of each site index class is 16.3 meters (Figure 2)

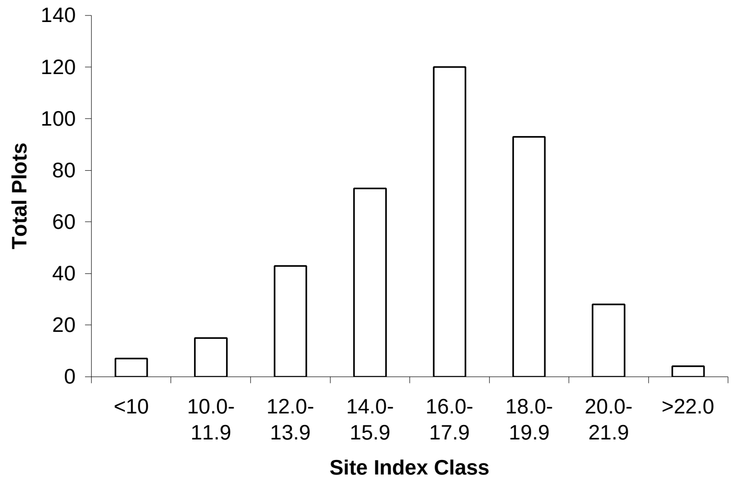


Figure 2. The distribution of the 383 Northern Ontario jack pine plots by site index class used in the Carmean et al. (In Press) study.

The height model (Carmean et al. (In Press) is based on the Newnham (1988) revised form of the Ek (1971) equation and has the following form:

$$H_t = 1.3 + 4.1459\,(SI - 1.3)^{0.62244}\left(1 - K^{\frac{BH_{age}}{50}}\right)^{1.3723\,(SI - 1.3)^{-0.0802}} \tag{1}$$

where

$$K = 1 - \left(\frac{SI - 1.3}{4.1459\,(SI - 1.3)^{0.6224}}\right)^{\frac{1}{1.3723\,(SI - 1.3)^{-0.0802}}}$$

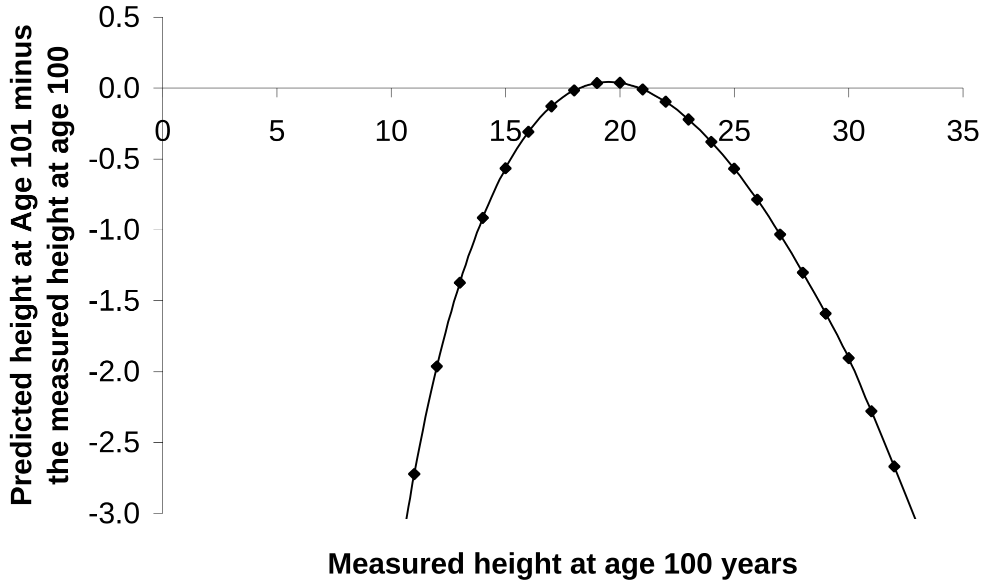
Where H_t is the mean dominant height, BH_{age} equals breast height age, and SI is the site index (base age equals 50 years). This is a polymorphic model that constrains the curves to pass through the site index at the specified base age. The height model is not solvable for site index, therefore Carmean et al. (In Press) developed an equation, which is based on Monserud's (1984) site index model and has the following form:

$$SI = b_0 + b_1 H_t + b_2 \ln(H_t) + b_3 \ln(BH_{age}) + b_4 \ln(BH_{age})^2 + b_5 \left(\frac{H_t}{BH_{age}} \right) + b_6 BH_{age} \ln(H_t) \quad (2)$$

The site index equation parameters (Carmean et al. In Press) are: β_0 = 13.1440, β_1 = 0.4050, β_2 = 7.4013, β_3 = -2.4941, β_4 = -1.1382, β_5 = -0.00185, and β_6 = 0.0240. The height and site index models are recommended for a spectrum of site indices, heights, and ages. Several scenarios were examined using the height model to generate pseudo-data as an illustrating example of inventory prediction. It is assumed that a series of stands are inventoried with no prior measurements of the site index. Then the performance of the height and site index models is assessed using the height predictions. Several scenarios were considered using the improved equations to assess the overall height predictions.

Example of Heights Measured at Breast Height Age 100

Suppose that several of the inventoried stands are at breast height age (BH_A) 100 and each of these stands has a different height. Furthermore, suppose that heights range from 10.0-40.0 m and the site indices are unknown. To predict future heights using equation 1, site index was predicted first using equation 2. The heights were then projected for each stand using the stand inventory data and the predicted site index. The projected heights should increase relative to the age 100 inventoried heights. But, except for the 19.0 and 20.0 m heights, all other inventoried heights project a decline in height from age 100 to 101 (Figure 3). The 19.0 and 20.0 m height classes have positive growth from age 100 to 101, and correspond approximately to site indices of 14.0 and 15.0 m, respectively. As described in Carmean et al. (In Press), the 14.0 and 15.0 m site indices correspond to about 36 plots. Thus, approximately 9.4% of the total plots (36 out of 383 total plots) are behaving positively with respect to height growth from age 100 to age 101 for these inventoried stands. Although these two height classes are "growing" from age 100 to 101, the height growth corresponding to the 19.0 and 20.0 m height classes are likely to be substantially underestimated. The declining heights imply that the site index equation is under-predicting the "true" site index.



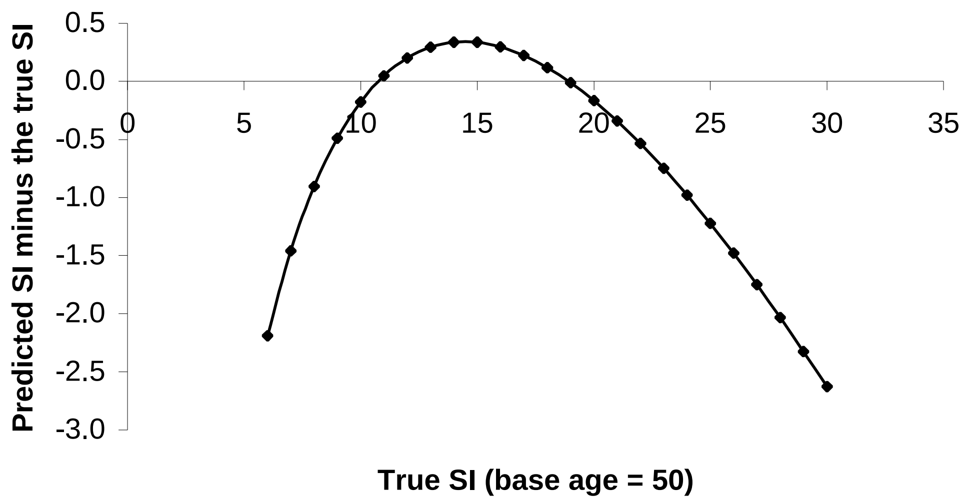


Figure 3. The age 101 projected bias that results from predicting the site index using the inventoried breast height age 100 stands and then predicting the age 101 height (top graph). The site index equation bias that results from predicting the site index given the breast height age 50 height (lower graph).

As discussed, the average site index is approximately 16.3 m for the Carmean et al. (In Press) study. The mean site index corresponds approximately to an age 100 21.5 m height. The inventoried heights of 21.0 and 22.0 m at age 100 forecast decreasing heights at age 101 (Figure 3). Although the decrease is not dramatic, it does reveal that for the approximate mean site index class it still predicts a decrease in age 101 height relative to age 100. As the inventoried heights diverge from the 19.0 and 20.0 m height classes, the decrease in height from age 100 to 101 is substantial, i.e. for an inventoried age 100 height of about 30.0 m, there is about a 2.0 m decrease in the height from age 100 to 101. Some of these inventoried height classes needed approximately 35 years before the projected height is higher than it was at age 100.

Example of Heights Measured at Breast Height Age of 20

Suppose that some of the inventoried jack pine stands in Northern Ontario are BH_A 20 and have heights of 4.0, 12.0, and 15.0 m. The predicted site indices for the inventoried BH_A 20 4.0, 12.0, and 15.0 m heights are 8.0, 19.9, and 22.9 m, respectively. All three of these inventoried stands are forecasted yearly to age 50 using the height model (Table 1). The forecasts reveal that from age 20 to age 21, the height has decreased for the inventoried heights of 12.0 and 15.0 m. The 12.0 m height declines over 2% from age 20 to 21, and the 15.0 m height declines almost 7.5%. The inventoried age 20 12.0 m height doesn't increase in height relative to the age 20 height until age 22. The inventoried age 20 15.0 m height predicts a decrease in height from age 20 to age 21 of almost 1.6 m, and hasn't become taller than the inventoried height until age 24. The 4.0 m mean height class over-predicts, i.e., it decreases in height from age 19 to age 20. Thus, the inventoried age 20 height of 4.0 m has to be backdated to age 18 before the predicted height (3.9 m) is less than the age 20 height. These inventoried age 20 stands tend to under and over-predict the site index for the higher and lower height classes, respectively.

Table 1. The projected heights for three stands with inventoried heights of 4.0, 12.0, and 15.0 m at age 20. The site indices of these stands were predicted first, then the heights were projected using the predicted site indices.

Age	Predicted Height		
	Site Index Class (m)		
	8.0	19.9	22.9
19	4.10		
20	4.00*	12.00*	15.00*
21	4.39	11.72	13.88
22	4.53	12.12	14.33
24	4.82	12.88	15.20
26	5.10	13.60	16.02
.	.	.	.
.	.	.	.
.	.	.	.
48	7.79	19.51	22.47
50	8.00	19.90	22.87

* Inventoried age 20 heights.

Example of Heights measured at Breast Height age 50

Suppose that some of the inventoried stands are at BH_A 50. The site index model might be expected to predict the same height, i.e., the measured heights. because by definition the site index is equal to the BH_A 50 height. But the predicted site index only equals the measured height at age 50 for two heights (Figure 3). Let's suppose that age 50 measured height is 16.0 m, i.e., the "true" site index is 16.0 m. Then the site index predicted using equation 2 is 16.3 m. Hence, the predicted site index equals the measured height at age 50 ("true" site index) if and only if the site index is approximately 10.7 or 19.0 m. The site index is overestimated within the site index interval of 10.7 and 19.0 m and underestimated outside this interval (Figure 4). The distribution of the 383 jack pine plots (Figure 2) illustrates that for the majority of their plots, the site index is being overestimated. At the data extremes, the deviations are approximately -1.5 and -0.5 m for the 7.6 and 22.4 m site indices, respectively.

Example of Heights Measured at Different Ages for Low and High Sites

Suppose there are heights that were inventoried at various ages, which have site indices of 10.0, and 20.0 m. The simulations were computed executing the following steps.

1) compute heights (equation 1) ($H_{t(1)}$) using the two site indices for ages 20-100 years (five year increments),
2) treat the computed heights as inventory records, and use the inventoried values to estimate site index using equation1,
3) use the estimated site indices to predict the heights ($H_{t(2)}$) at their respective inventory ages and,
4) calculate the height bias as $H_{t(1)}$ - $H_{t(2)}$.

If the site index estimates were relevant to the height from which they were estimated, then the heights predicted from them would equal their actual values. Then the height-age curves would be identical for all measurement ages chosen to predict site index, i.e., there should be one master height-age curve for each site index value. The generated height-age curves and corresponding bias vary depending upon which height measurement age combination is used to predict the site index. (Figure 4). The predicted height-age curves are more consistent for the 20.0 m site index. The height-age curves generated using the site index of 10.0 m exhibit a high amount of variability and the variability increases as a function of time. This implies that the site index equation is performing worse for lower sites. The bias that results from predicting height using different measurement ages to predict site index (Figure 4) would be zero if they were unbiased, i.e., the curves would lie horizontal along the x-axis. As illustrated, for the true site index of 10.0 m, the profile line is almost unbiased for the breast height age of 25. If our measurement age is younger than 25 then the site indices are being overestimated and if the measurement age is older than 25, then the site indices are being overestimated.

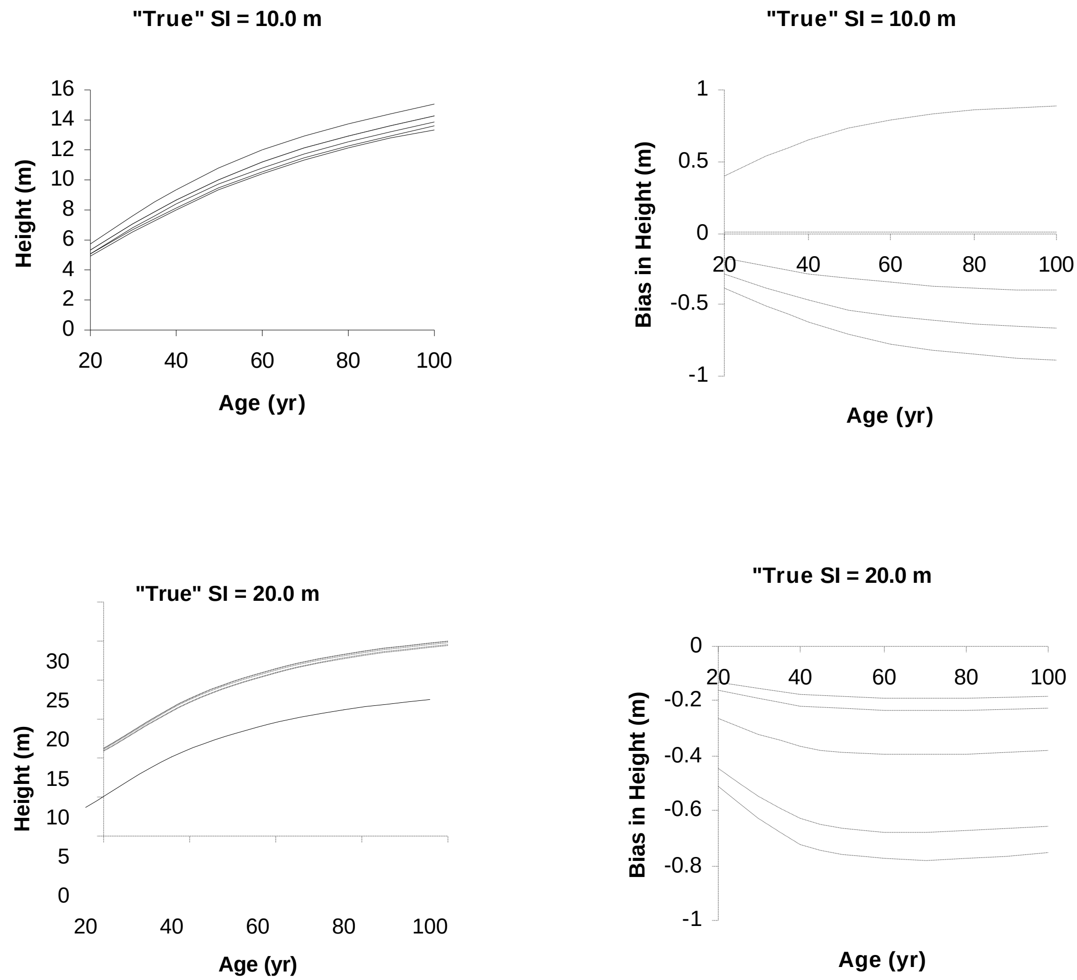


Figure 4. An example of height-age curves generated using predicted site indices from different height-age combinations that belong to the single master site index curves of 10.0 and 20.0 m. The corresponding bias in height that results from using the different height-age combinations of the master site index curves of 10.0 and 20.0 m.

The predicted heights at age X (X = 20, 40, 60, 80, and 100) using different measurement ages to predict site index given the true site indices of 10.0 and 20.0 m are plotted against the measurement age (Figure 5). These profiles would be horizontal if the site index estimation were consistent to the values of height. Figure 5 illustrates that the heights predicted depend upon what measurement age is chosen to predict site index. The predictions using the true site index of 10.0 m performs the worse, i.e., there is more variation for the predicted heights given the measurement age when using the 10.0 m site index. The profiles exhibit a quadratic trend for the 20.0 m site index. The bias that results from the predicted height at age X given the age of measurement for both site indices 10.0 and 20.0 m are presented in Figure 6. The predicted heights at age X tend to be overestimated when using young measurement ages to predict the site index and overestimated when using measurement ages greater than approximately 40 for the true site index of 10.0 m. For the true site index of 20.0, the least bias is present in the interval of approximately measurement ages 40-60. Outside this interval the bias increases as the measurement age diverges. This scenario reveals that there can be substantial bias when using a system of incompatible site index and height models. Several alternatives were explored to reduce the bias that results when using separate site index and height equations.

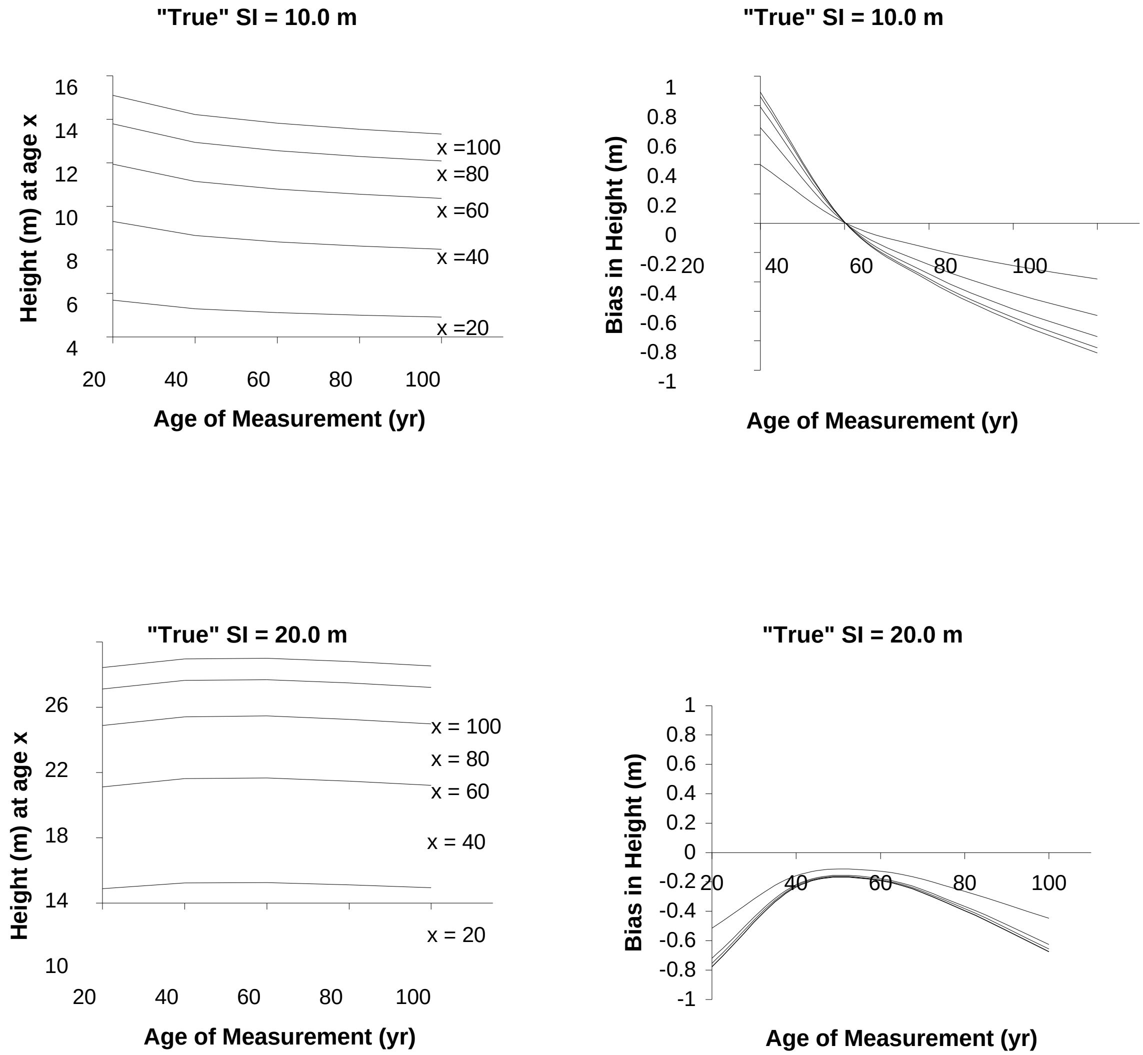


Figure 5. The predicted heights for the measurement ages of 20, 40, 60, 80, and 100. Which are generated from the height-age curves that have true site indices of 10.0 and 20.0 m. The bias corresponding to the predicted heights for the true site indices of 10.0 and 20.0 m. If the site index and height published models were compatible, then each of the profiles would be horizontal.

Alternatives to Reduce Bias

There are two methods used to reduce the bias in height predictions. Both methods assume that the height model (equation 1) does not need improvement. The first method assumes that the primary objective is to predict height. The site index model (equation 2) is used to predict site index when this information is unknown for use in the height model. Hence, the site index model (equation 2) parameters are re-estimated by minimizing errors in height predictions while holding the published height model parameters constant using pseudo-data generated from the height model (equation 1). The second method uses dynamic equations as an alternative to the fixed base-age concept. There are two dynamic equations that are considered. The first is a published dynamic equation for lodgepole pine (Cieszewski and Bella, 1989), and the second is to fit a dynamic equation to the pseudo-data. The "height optimizing SI" and "expanded height optimizing SI" models were assessed using the pseudo-data and compared to the published model for height predictions. In addition, a random inventory scenario was constructed to illustrate the performance for all the considered models.

Estimating the Site Index Parameters in the Height Model

The site index model (equation 2) parameters were re-estimated by inserting the site index model into the height model (equation 1) and minimizing the error sums of squares with respect to height. Thus, the "height optimizing" site index model has the following form.

$$H_t = 1.3 + 4.1459\,(g(H_t, Age) - 1.3)^{0.62244}\left(1 - K^{\frac{BH_{age}}{50}}\right)^{1.3723\,(g(H_t, Age) - 1.3)^{-0.0802}}$$

where

$$K = 1 - \left(\frac{g(H_t, Age) - 1.3}{4.1459\,(g(H_t, Age) - 1.3)^{0.6224}}\right)^{\frac{1}{1.3723\,(g(H_t, Age) - 1.3)^{-0.0802}}}$$

and

$$g(H_t, Age) = b_0 + b_1\, H_t + b_2\, ln(H_t) + b_3\, ln(BH_{age}) + b_4\, ln(BH_{age})^2 + b_5\left(\frac{H_t}{BH_{age}}\right) + b_6\, BH_{age}\, ln(H_t) \quad (2)$$

This site index parameter estimation technique used pseudo-data from the height model (equation 1) based on site indices systematically chosen from 8.0-22.0 m (2.0 m increments) and BH_A 20-100 (5 year increments). Hereafter, the pseudo-data for height ($H_{t(1)}$) are referred to as the true heights. The "height optimizing SI" model site index parameters were estimated by holding the Carmean et al. (In Press) height model published parameters constant. The "height optimizing SI" model and the published Carmean et al. (In Press) site index estimated parameters are presented in Table 2. The parameters for the "height optimizing SI" model are substantially different from the published parameters. The site indices were then predicted for BH_A 20-100 (5 year increments) and the true height combinations. These predicted site indices for both the "height optimizing SI" and published models were used to predict height ($H_{t(2)}$).

Table 2. The estimated parameters for the published and "height optimizing SI" site index models.

Parameter	Published	"Height optimizing SI"
β_0	13.1440	-4.3101
β_1	0.4050	0.6752
β_2	7.4013	-2.2446
β_3	-2.4941	7.1776
β_4	-1.1382	-1.4397
β_5	-0.00185	21.6874
β_6	0.02400	0.01956

Systematic bias was observed for the "height optimizing SI" model by plotting the ratio of the true site index to the predicted site index against age and height. An easy solution to reduce this bias was to regress the site index ratio against height and age. Thus, the site index ratio was regressed on age and height. This site index ratio model has 13 parameters and an r-square greater than 0.96. To adjust for the systematic bias, the ratio is predicted as a function of age and height. Then the site index ratio is multiplied against the predicted "height optimizing SI" model site index. Once this "expanded height optimizing SI" model site index is computed the heights are predicted.

The average deviation, mean absolute deviation, SSE, and RMSE were computed using the true and predicted heights for the published, "height optimizing SI", and "expanded height optimizing SI" site index models (Table 3). The mean deviation is more favorable using the "height optimizing SI" model and the mean deviation when using the "expanded height optimizing SI" model is negligible. Both the "height optimizing SI" and "expanded height optimizing SI" models perform substantially better than the published model. The error sums of squares has been reduced from 52.25 (published model) and 3.52 ("height optimizing SI" model) to 0.20 for the "expanded height optimizing SI" model, which results in a RMSE of 0.03986. In practical terms, the SSE for the published model results in a standard deviation of over 0.63 m (about 2.0 ft), while the "expanded height optimizing SI" model results in a standard deviation of approximately 0.04 m (1.6 in.).

Table 3. The mean deviation, mean absolute deviation (MAD), error sums of squares (SSE), and root mean square error (RMSE) summary statistics using the published, "height optimizing SI" and "expanded height optimizing SI" models.

Criterion	Published	"Height optimizing SI"	"Expanded height optimizing SI"
Mean Deviation	0.2560	-0.0002753	-0.0005751
MAE	0.4278	0.1278	0.02980
SSE (Ht)	52.25	3.52	0.20
RMSE	0.6364	0.1652	0.03986

The bar charts (Figure 6) illustrate the bias reduction when using both the "height optimizing SI" and "expanded height optimizing SI" models. The mean deviation for the "height optimizing SI" and "expanded height optimizing SI" models are about equal for the spectrum of site index classes, but both are substantially smaller than the mean deviation using the published model. The mean absolute deviation is more favorable when using the "expanded height optimizing SI" model, except for the 8.0 m site index class where it performs about equally with the "height optimizing SI" model. The SSE by site index class illustrates that the "expanded re-estimated SI model" performs substantially better than the other two models. In fact, the SSE are negligible, hence they fail to register for most site indices.

The simulations using the published site index and height models that generated the height-age curves and corresponding bias graphs (Figure 4), and the height at age X versus measurement age and their respective bias graphs (Figure 5) for the site indices 10.0 and 20.0 m were conducted using the "expanded height optimizing SI" site index model. The site indices 10.0 and 20.0 m height-age curves (Figure 7) reveal that there appears to be almost the one expected master curve for each of these site indices. Furthermore, the resulting bias for the two site indices is seen to be negligible and is dramatically reduced relative to the bias using the published site index model. The predicted heights using different measurement ages to predict site index are almost horizontal (Figure 8) and the corresponding bias for both site indices are negligible. The 20.0 m site index class appears to be performing poorly but the largest absolute bias is approximately 0.15 whereas for the published site index model it is 0.80, a reduction of over five times.

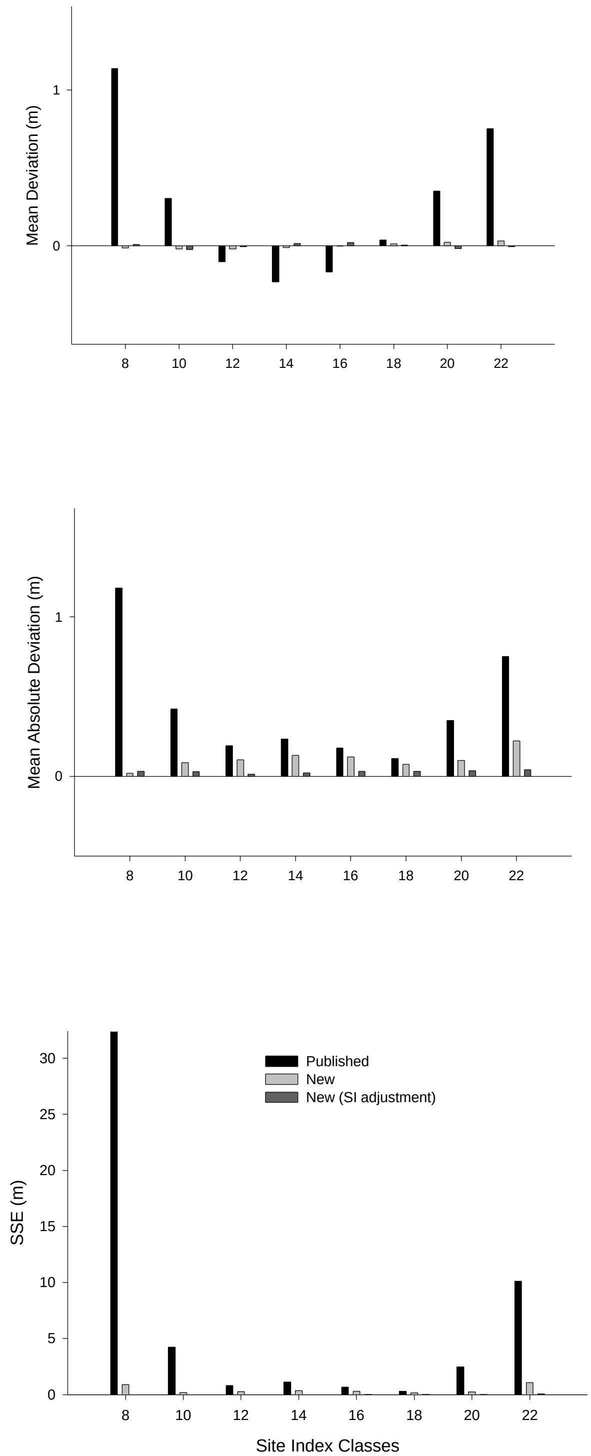


Figure 6. The mean deviation, mean absolute deviation, and SSE by site index class for the published, "height optimizing SI", and "expanded height optimizing SI" models.

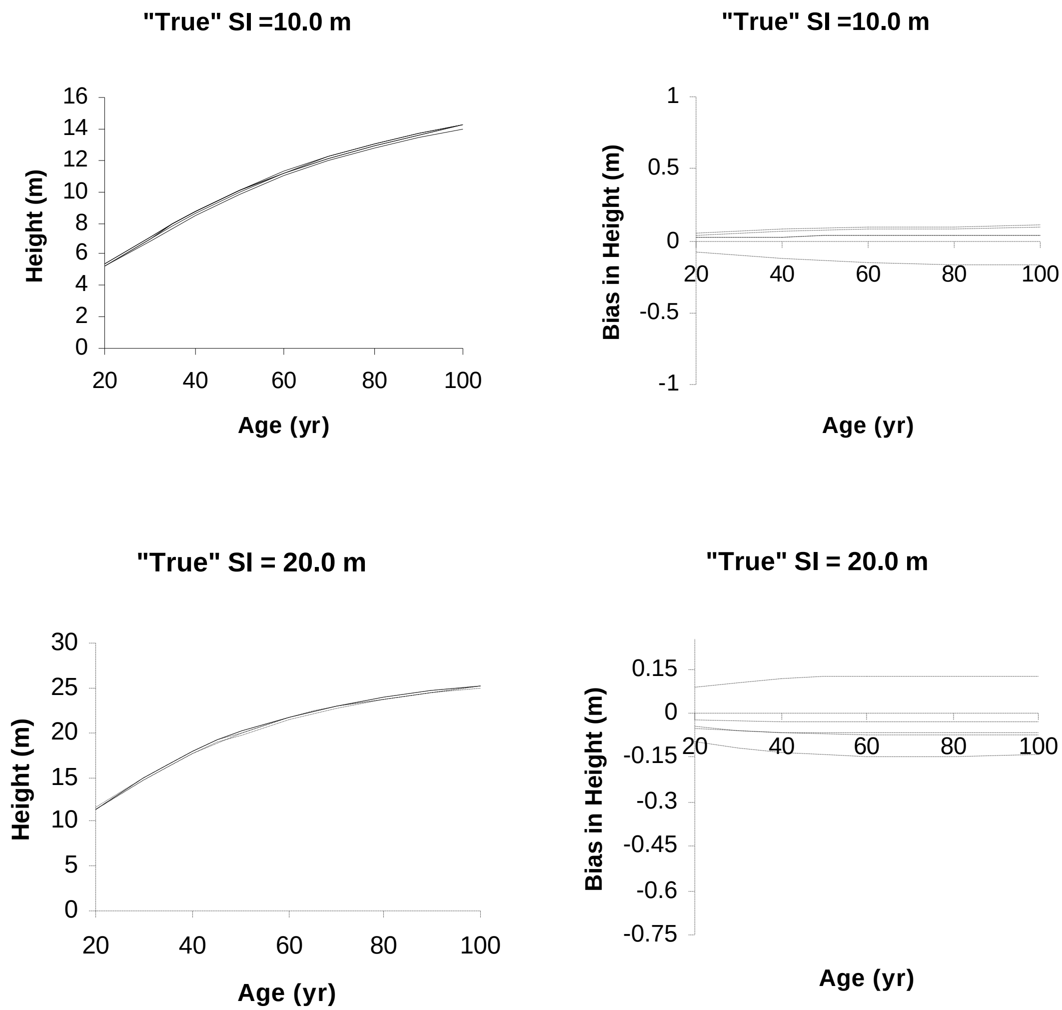


Figure 7. An example of height-age curves generated using the "expanded height optimizing SI" model to predict site indices from different height-age combinations that belong to the single master site index curves of 10.0 and 20.0 m. The corresponding bias in height that results from using the different height-age combinations of the master site index curves of 10.0 and 20.0 m.

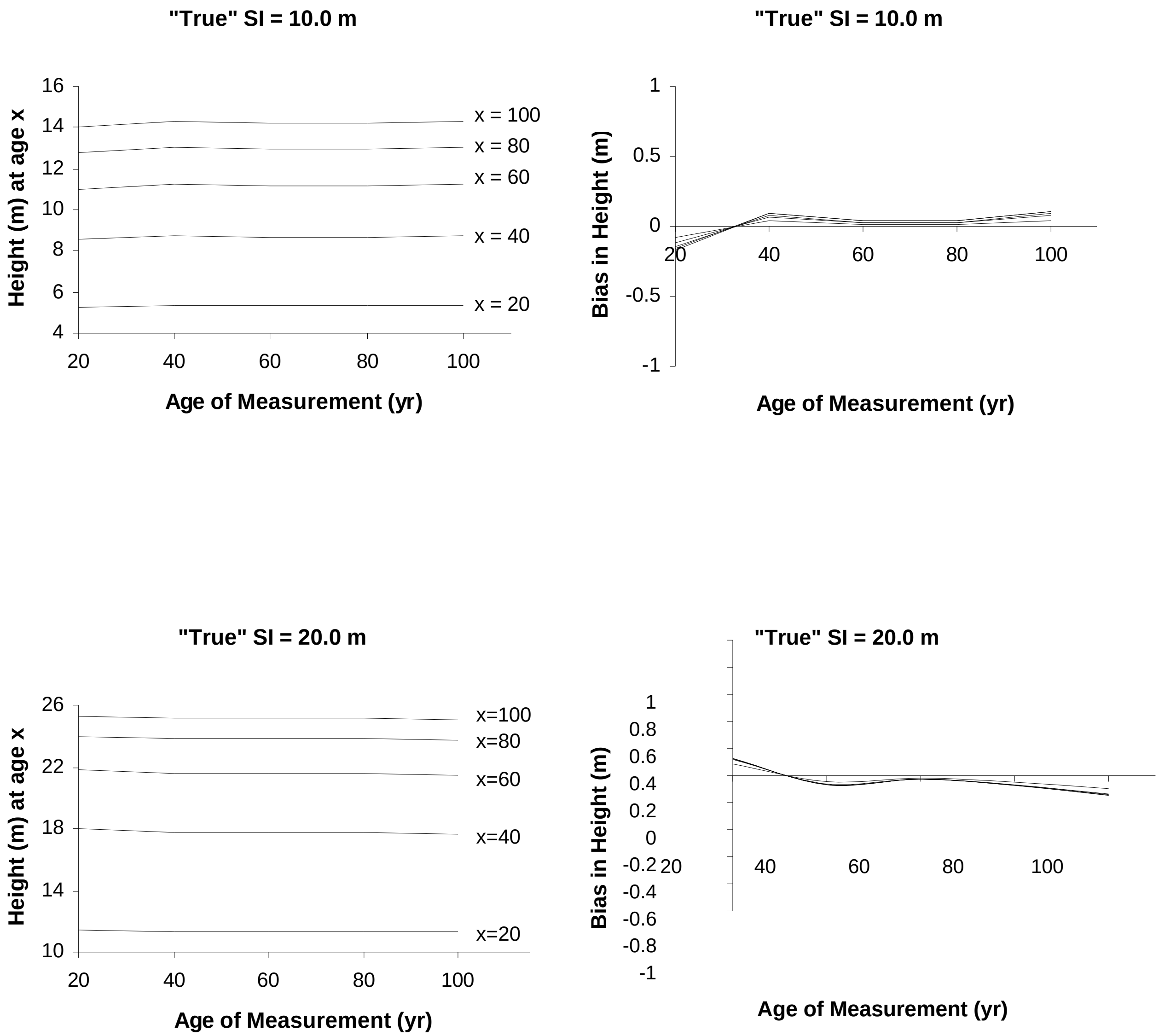


Figure 8. The heights predicted for the measurement ages of 20, 40, 60, 80, and 100 using the "expanded height optimizing SI" model. Which are generated from the height-age curves that have true site indices of 10.0 and 20.0 m. The bias corresponding to the predicted heights for the true site indices of 10.0 and 20.0 m. If the "expanded height optimizing SI" and height models were compatible, then each of the profiles would be horizontal.

Dynamic Equations

It was expected that dynamic equations would provide superior estimations in computing inventory projections. For illustrative purposes, the lodgepole pine dynamic equation by Cieszewski and Bella (1989) was used "as is" without re-estimating the model parameters for the jack pine conditions. The model has the following form:

$$H(t, h_x, x) = \frac{h_x + d + f}{2 + \dfrac{80b\ t^{-1-a}}{h_x - d + f}}$$

where

$$f = \sqrt{(h_x - d)^2 + 80b\ h_x x^{-1-a}}$$

and

$$d = 20b\,50^{-1-a}$$

The published estimated parameters are α = 0.2089 and β = 100.566, h_x is the initial height with age = x, and the model can predict the height at any other time t.

As a recent example, a more advanced dynamic equation (Cieszewski 2001) was fitted to the same psuedo-data generated earlier for the site indices of 8.0-22.0 by 2.0 m increments and the BH_A are 20-100 (5 year increments). The parameters were fitted using the dummy variable approach described in Cieszewski et al.(In Review), i.e., treating the true site indices as local parameters while estimating the global parameters. The dynamic height model has the following form:

$$H(h_0, t_0, t) = h_0 \left(\frac{t}{t_0} \right)^{u+d} \left(\frac{t_0^d f + k}{t^d f + k} \right)$$

where

$$f = t + \left(t^2 + \frac{2kh_0}{t_0^{u+d}} \right)^{\frac{1}{2}}$$

and

$$t = \left(\frac{h_0}{t_0^u} - h \right)$$

where h_0, and t_0 are the initial observed height and age, or where site index and the corresponding base age are ordinarily. The site indices were treated as parameters, thereby not forcing the curve through an arbitrary reference point. The estimated global parameters using this approach are δ=1.416274, η= 589.8711, κ= 399325.39, and ν= -0.5946.

These are dynamic models and hence, the bias when $t = t_0$ is zero. Thus, it is known both of these dynamic models will outperform the published, "height optimizing SI", and "expanded height optimizing SI" models for the present inventory, but our interest is in updating or predicting the future inventory.

An Illustrative Example of Inventory Projections

A simulation was conducted to demonstrate the liability of the considered models to inventory projections. The inventory updates using the published, "height optimizing SI', and "expanded height optimizing SI" site index models in conjunction with the published height model, and using the lodgepole pine and jack pine dynamic models was conducted using the following steps. Site indices were systematically chosen from 8.0-22.0 by 2.0 m increments. Then the initial age was randomly selected over the interval 5 and 20. Using the randomly selected age as the baseline, six additional age classes were selected by adding 20, 40, 60, 80, 100, and 120 years to the randomly selected age. The heights were then projected for each of these age and site index class combinations for one, five, and ten-years. The site indices and age combination were used in the height model to produce the actual inventory. The performance of the four models was evaluated by comparing the mean deviation, mean absolute deviation and SSE for the one, five, and ten-year inventory updates (Table 4).

Table 4. The projected heights mean deviation, mean absolute deviation (MAD), and error sums of squares (SSE) from the example of a random one, five, and ten-year inventory updates. The heights were projected using the published site index, "height optimizing SI", "expanded height optimizing SI", lodgepole pine dynamic, and jack pine dynamic models

Criterion	Published	"Height optimizing SI"	"Expanded height optimizing SI"	Lodgepole Pine Dynamic	Jack Pine Dynamic
			One-year Inventory Projection		
Mean Deviation	-0.4385	0.0226	0.0691	0.0501	-0.00132
MAD	0.5854	0.1958	0.1294	0.0501	0.00674
SSE	39.4836	3.9329	1.6785	0.2459	0.0132
			Five-year Inventory Projection		
Mean Deviation	-0.4378	0.0288	0.0610	0.2473	-0.00141
MAD	0.6037	0.2047	0.1389	0.2473	0.03060
SSE	40.8199	4.3039	2.0943	5.8876	0.2190
			Ten-year Inventory Projection		
Mean Deviation	-0.4362	0.0357	0.0520	0.4835	0.00534
MAD	0.6228	0.2144	0.1490	0.4835	0.05668
SSE	42.4629	4.8367	2.6953	21.7502	0.6193

The inventory updates scenario reveals that the jack pine dynamic model performs substantially better than the other models for all criteria. The jack pine dynamic model reduces the projected heights error sums of squares for the one and ten-year inventory projections by almost 3000 and 69 times from the published site index model, respectively. The jack pine dynamic model performance gain is reduced as the length of the inventory forecast increases, but it still performs substantially superior to the published site index model. The lodgepole pine dynamic model performs substantially better than the published site index model, especially for the one-year inventory projection. It exhibits systematic bias that could be adjusted, and hence improve the performance for the ten-year inventory projection. The "expanded height optimizing SI" model performs more favorably than the published site index model for all inventory projections and has little variability between the inventory projections. But the jack pine dynamic model performs substantially better than the other models and reduces the error sums of squares by over 4.3 times relative to the "expanded height optimizing SI" model.

The one, five, and ten-year inventory projections for the "height optimizing SI" and lodgepole pine dynamic models are presented in Figure 9. The inventory projections for the published site index (top graph) and jack pine dynamic (bottom graph) models are presented in Figure 1. The published site index model illustrates trends that were evident in earlier analysis, i.e., there is evidence of declining heights. The published model predicts smaller heights for some ages, even for the ten-year inventory update. The "height optimizing SI" model exhibits some declining heights for the older age and higher site index combinations. The "expanded height optimizing SI" model did not exhibit any decreasing heights for any of the inventory projections. The lodgepole pine dynamic model does exhibit systematic overestimated bias for all inventory projections. The jack pine dynamic model exhibits the least bias and for most of the inventoried heights, it is difficult to distinguish the projected and inventoried heights.

The randomly simulated 10-year inventory projection bias for the published and "expanded height optimizing SI" site index models, and the lodgepole pine and jack pine dynamic models are illustrated in Figure 10. The published site index model performs poorly relative to the other models. The lodgepole pine dynamic model exhibits systematic bias but still performs better than the published site index model for these 1, 5, and 10-year inventory predictions. The jack pine dynamic model has substantially less bias for all the age classes relative to the other three models.

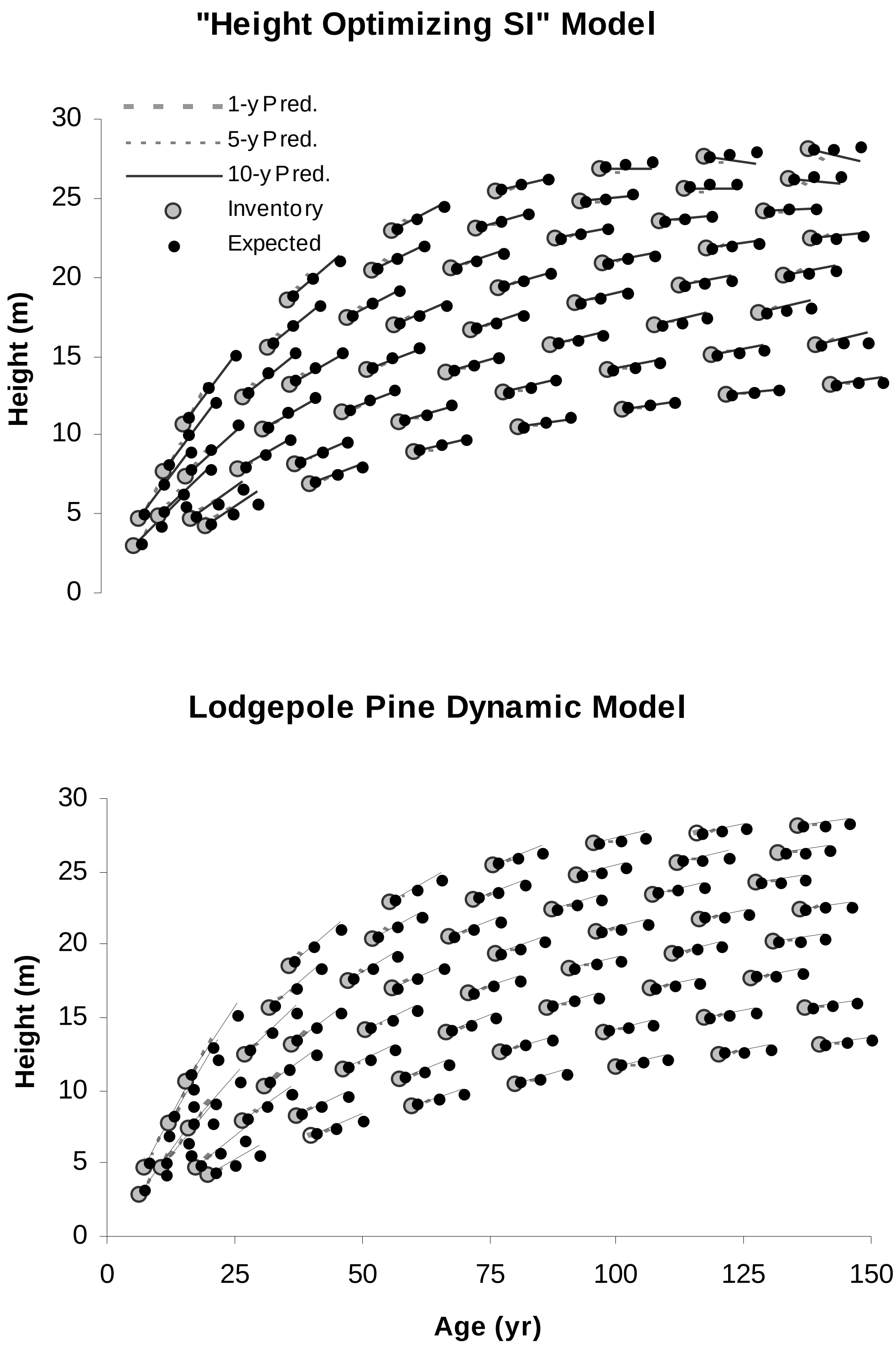


Figure 9. The example of the randomly generated one, five, and ten-year inventory projections using the "height optimizing SI' and lodgepole pine dynamic models.

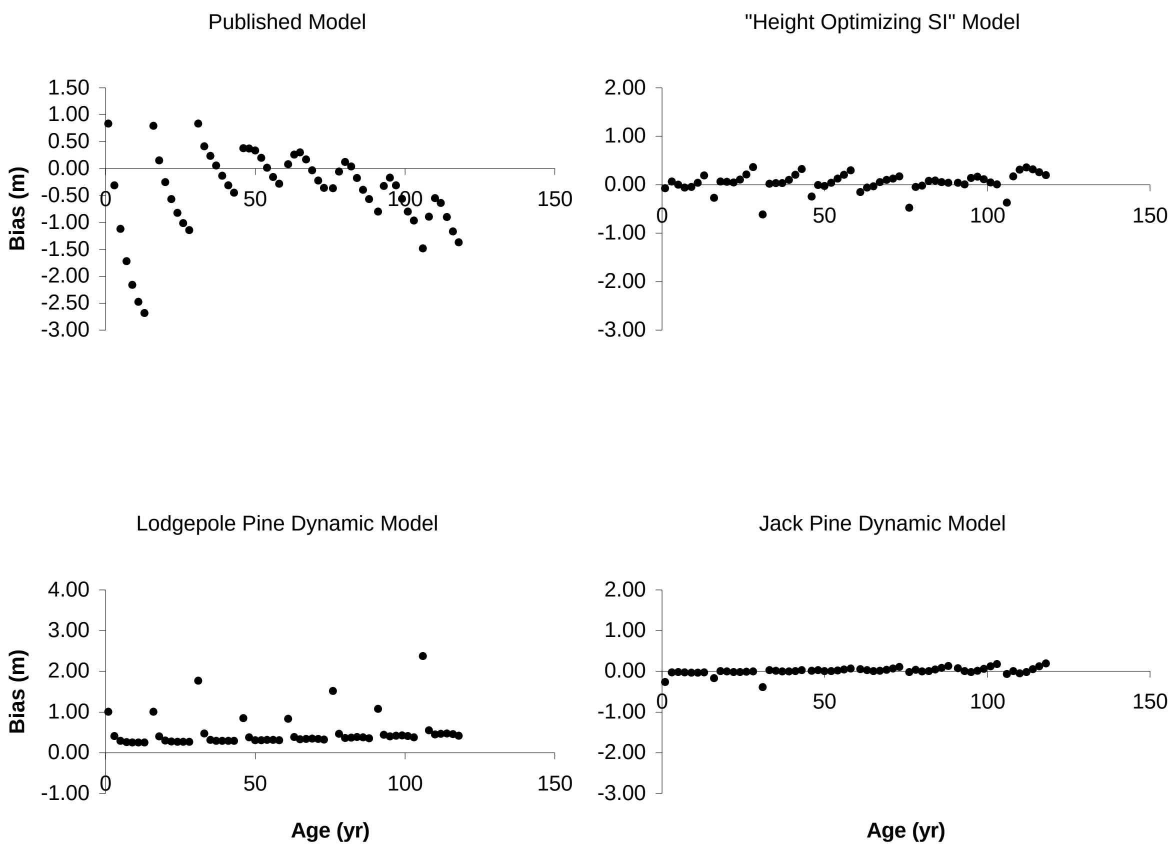


Figure 10. An example of the ten-year random inventory update height bias using the published and "height optimizing SI" site index models with the published height model, and the lodgepole pine and jack pine dynamic height models.

Conclusion

The bias in height predictions that results when using an incompatible system of height and site index models can be substantial. Re-estimating the site index model parameters by optimizing with respect to height substantially reduced the height prediction deviations. The bias in height projections using the published, "height optimizing SI", and "expanded height optimizing SI" site index models occurs because of the incompatibility between the site index models and the height equation. Because of this incompatibility, there may be additional bias induced by the height equation. When projecting the heights from the true site index, there is a slope associated with the true site index and projection period. Since the predicted site index is typically different from the true site when using the incompatible site index models, there may be a different slope for the projection period, hence another source of bias is induced. The inventory projections using the "height optimizing SI" model illustrated that the bias in height projections are substantially reduced relative to the published site index model, but since the incompatibility exists it is possible to project decreasing heights. The superiority of dynamic equations is demonstrated by the lodgepole pine dynamic equation that performs well for the jack pine height model. The jack pine dynamic model demonstrated that not only can it produce more accurate predictions but that the model is more parsimonious. By definition, the dynamic equations predict the measured height for any inventoried age. Therefore, the inventory projection height bias for the dynamic equations is caused by the different slopes for these projection periods from the published height model. The change in slope will generally be similar for most species for short inventory projections, which is why the lodgepole pine model performs adequately and the jack pine dynamic equation performs substantially better for these inventory projections. The published system of site index and height equations has a total of 11 parameters, whereas the jack pine dynamic equation has four parameters. In addition, the jack pine dynamic model has the capability to express both the height and site index models by a single equation. The dynamic height model gains its flexibility from its inherent ability to depict the three dimensional relationship between height, age and the initial conditions.

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