A Method for Calculating the Internal Rate of Return on Marginal Silviculture Investments

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Abstract

As with most investment managers, foresters are regularly faced with quantifying the impact of expenditures on the financial performance of the asset they manage – timber and timberland. The most common measures of financial performance are net present value and internal rate of return. Internal rate of return is the more common measure while net present value is generally preferred by academics when making such financial decisions. In many, if not most cases, foresters will calculate both measures for use in the investment analysis. However, there do exist small, but at times meaningful, differences in the resulting decisions based upon the investment criteria used. Additionally, there are cases where the investors and managers are interested in quantifying the marginal impact of each silvicultural treatment being evaluated in the capital budgeting process. We explore methodology to calculate the internal rate of return on these marginal investment decisions in a plantation forestry context.

Introduction

For many years foresters have drawn on basic finance methods and principles when evaluating investments in timber and timberland. There exist many basic forest economics and timber management texts that discuss such investment criteria as net present value, internal rate of return, or benefit / cost ratio and their use in timber management situations. In 1849 a German forester, Martin Faustmann developed the concept of a continuing series of rotations (investments) to address the time horizon issues when using net present value in a forestry context. This work still stands today as the proper method to address such issues across all types of investment questions where differing time horizons are encountered (Davis et al 2000).

Most businesses have a formal capital budgeting process that evaluates investment alternatives using a variety of commonly accepted financial criteria. Financial attributes that relate to both the expected return and the risk of the specific investment are evaluated and projects are funded based upon these attributes. Considerable survey research ahs been performed to identify the most commonly used capital budgeting techniques of corporate America (Pike, 1996). For many years payback and internal rate of return were the most often used investment criteria even though these methods were considered inferior to net present value by finance academics (Aggarwal, 1980). However, more recently there has been a shift to using net present value and internal rate of return in the capital budgeting process (Ryan, 2002). Firms investing in timber and timberland have followed these same trends in capital budgeting methodology traditionally using payback and IRR as the primary measures. Recently both IRR and NPV have become the standard criteria.

A common practice in capital budgeting is to evaluate marginal or additional investments in such real assets as manufacturing plants where these additional investments contribute to increased productivity or output. It is customary to compute the internal rate of return on this marginal investment (Gitman, 1982). In timber and timberland applications the marginal investment is usually additional silviculture treatments and the expected results are increased growth rates and additional timber revenue. This has become a standard method for evaluating silviculture expenditures. However, the calculation of these IRRs is not straightforward due to the impact of the treatments on stand development and ultimately selection of optimal rotation age. Such issues have generally been addressed by requiring a common rotation age for all regimes evaluated and ignoring situations where optimal rotation age differs by silviculture treatment.

Analysis

We derive here a method to use when calculating the internal rate of return of marginal silviculture investments in an even-aged plantation setting. The concept can easily be expanded to other timber investment situations. Consider a simple plantation example represented by the cash flow time line below:

Costs	-R	-R	-R . . .
Year	0	t	2t . . .
Revenues		$\mathbf{SY}_t$	$\mathbf{SY}_t$ . . .

Where R = Regeneration costs
$\mathbf{S}$ = A row vector of stumpage prices by product
$\mathbf{Y}_t$ = A column vector of yields by product
$\mathbf{SY}_t$ = Gross revenue from timber harvest
t = Optimal rotation age

This is the base case "untreated" regime and the bare land value (land expectation value) is computed as:

$$BLV_{UT} = \frac{-R(1+i)^t + SY_t}{[(1+i)^t - 1]} \tag{1.1}$$

Where i = the real discount rate

Now, consider a case where some additional amount D (dollars) is spent at planting to produce some additional amount of yield by product (hence, a column vector of yields by product) that are represented by $\mathbf{Y}_{mt}$ – the marginal yield by product associated with the additional silviculture treatment at rotation age t. Further assume that the optimal rotation age does not change when this marginal treatment is utilized so both the treated and untreated stands have an optimal rotation age of t. The cash flow time line can be represented as:

Costs	-(R+C)	-(R+C)	-(R+C) . . .
Year	0	t	2t . . .
Revenues		$\mathbf{S}(\mathbf{Y}_t+\mathbf{Y}_{mt})$	$\mathbf{S}(\mathbf{Y}_t+\mathbf{Y}_{mt})$ . . .

Where C = cost of the marginal silviculture treatment

Y_{mt} = the marginal yield (Y_{mt}) produced at time t
Other variables previously defined.

The net present value of this cash flow time line is represented by:

$$BLV_T = \frac{-(R+C)(1+i)^t + S(Y_t + Y_{mt})}{[(1+i)^t - 1]} \tag{1.2}$$

Now, to calculate the marginal IRR of the silviculture treatment costing C dollars in year 0 and producing SY_{mt} marginal revenue in year t, we subtract $BLVT_{UT}$ from BLV_T, set this quantity to 0 and solve the resulting equation for i, the IRR for the marginal silviculture treatment.

$$\Delta BLV = \frac{-(R+C)(1+i)^t + S(Y_t + Y_{mt}) - [-R(1+i)^t + SY_t]}{[(1+i)^t - 1]} \tag{1.3}$$

Where $\Delta BLV = BLV_T - BLV_{UT}$

Upon some rearrangement.

$$\Delta BLV = \frac{-C(1+i)^t + SY_{mt}}{[(1+i)^t - 1]} \tag{1.4}$$

Setting ΔBLV to 0 and solving for i produces

$$i = \left[\frac{SY_{mt}}{C}\right]^{1/t} - 1 \tag{1.5}$$

This result is the standard method to calculate the IRR of a conventional cash flow stream with one cost (C) at the beginning of the investment and one revenue (SY_{mt}) at the end of the investment - i.e. a zero coupon bond. Hence, we start with the BLV formula for each regime and end up with a derivation of the internal rate of return for some additional contemplated silviculture investment at planting. An example might be utilizing seedlings of better genetic stock, a mechanical or chemical site preparation treatment, or possibly some type of fertilization at planting. This derivation provides a framework to follow for the more complicated situations presented below. This result is clearly the proper formula to use for our contrived example of equal rotation ages for both the treated and untreated stand.

Now, consider a more general case where some additional investment (C) is made at planting and some additional yield by product (**Y_{mt2}**) is received at rotation age t2 which can be either shorter or longer than the untreated regime with optimal rotation age t1. Again the untreated regime can be represented in a cash flow time line as:

Costs	-R	-R	-R . . .
Year	0	t1	2t1 . . .
Revenues		**SY**$_{t1}$	**SY**$_{t1}$ . . .

With a $BLV_{UT,t1}$ as:

$$BLV_{UT,t1} = \frac{-R(1+i)^{t1} + SY_{t1}}{[(1+i)^{t1} - 1]} \qquad (1.6)$$

Similarly the treated regime can be represented as:

Costs	-(R+C)	-(R+C)	-(R+C) . . .
Year	0	t2	2t2 . . .
Revenues		**S**(**Y**$_{t2}$+**Y**$_{mt2}$)	**S**(**Y**$_{t2}$+**Y**$_{mt2}$) . . .

With a $BLV_{T,t2}$ as:

$$BLV_{T,t2} = \frac{-(R+C)(1+i)^{t2} + S(Y_{t2} + Y_{m,t1})}{[(1+i)^{t2} - 1]} \qquad (1.7)$$

We assume that t1 and t2, the rotation ages for the untreated and treated regimes are optimal rotation ages given the assumptions made. For the marginal IRR to be computed in this way both regimes have to be at their individual economic optimum rotation ages. Otherwise the marginal IRR estimates will biased and will always be an underestimate of the true maximum IRR for the marginal silviculture treatment. Again, we calculate the ΔBLV by subtracting equation 1.6 from 1.7.

$$\Delta BLV = \frac{-(R+C)(1+i)^{t2} + S(Y_{t2} + Y_{m,t2})}{[(1+i)^{t2} - 1]} - \frac{-R(1+i)^{t1} + SY_{t1}}{[(1+i)^{t1} - 1]} \quad (1.8)$$

If equation 1.8 is set to zero, then it cannot be analytically solved for i, the marginal IRR for the silvicultural treatment(s) of interest. However, equation 1.8 can be restated as:

$$\frac{-(R+C)(1+i)^{t2} + S(Y_{t2} + Y_{m,t2})}{[(1+i)^{t2} - 1]} = \frac{-R(1+i)^{t1} + SY_{t1}}{[(1+i)^{t1} - 1]} \quad (1.9)$$

This implies that the marginal IRR occurs at the point where $BLV_{T,t2} = BLV_{UT,t1}$. This point, if it exists, can be easily found using numerical methods and "good" guesses will exist as a starting point based upon the values of $BLV_{T,t2}$ and $BLVU_{T,t1}$ at commonly encountered values of i, the real discount rate. Hence when $BLV_{T,t2} = BLV_{UT,t1}$ then this implies that i is the IRR for the marginal investment, C.

Now consider the most general form of this situation where multiple costs (C_js) and multiple revenues (D_js) occur throughout the cash flow time line – where j represents the year in which the cash flow occurs. A cash flow time line representing this situation for the untreated regime is:

Costs	C_0	C_4	C_{11}	C_{20}	C_0	C_4	C_{11}	C_{20}	C_0 . . .
Year	0	4	11	20	t1	t1+4	T1+11	T1+20	2t1 . . .
Revenues				D_{20}	D_{t1}			D_{20}	D_{t1} . . .

The $BLV_{UT,t1}$ can be written as:

$$BLV_{UT,t1} = \frac{\sum_{j=0}^{t1}(D_j - C_j)(1+i)^{t1-j}}{[(1+i)^{t1} - 1]} \qquad (1.10)$$

Similarly for the treated regime the cash flow time line might be represented as:

Costs	C_0	C_4	C_{11}	C_{20}	C_0	C_4	C_{11}	C_{20}	C_0 . . .
Year	0	4	11	20	t2	t2+4	T2+11	T2+20	2t2 . . .
Revenues				D_{20}	D_{t2}			D_{20}	D_{t2} . . .

The $BLV_{T,t2}$ can be written as:

$$BLV_{T,t2} = \frac{\sum_{j=0}^{t2}(D_j - C_j)(1+i)^{t2-j}}{[(1+i)^{t2} - 1]} \qquad (1.11)$$

As before subtracting 1.10 from equation 1.11 and setting to zero yields the same result:

$$\frac{\sum_{j=0}^{t2}(D_j - C_j)(1+i)^{t2-j}}{[(1+i)^{t2} - 1]} = \frac{\sum_{j=0}^{t1}(D_j - C_j)(1+i)^{t1-j}}{[(1+i)^{t1} - 1]} \qquad (1.12)$$

So when $BLV_{T,t2} = BLV_{UT,t1}$ then this is the marginal IRR of the additional investment. This investment might represent one specific silviculture treatment or possibly several evaluated as a “package” of treatments. In this form, there might exist no real marginal IRRs or possibly several marginal IRRs depending upon the nature of the cash flow stream. This is a common problem with IRR in nonconventional cash flow patterns. A sophisticated numerical analysis routine can identify whether any real roots exist or if multiple roots exist to equation1.12. Regardless of the numerical analysis routine chosen, most situations are quite easy to solve numerically and produce only one real solution making the answer unambiguous and easy to interpret. Additionally, equation 1.12 is easy to compute from basic information about the timber management regimes. The steps to compute the values of interest include:

1. Build an array for each case (treated and untreated) that contains year, cost in that year, and revenue in that year.
2. Compute the BLV for each treatment at some real discount rate, i.
3. If $BLV_{T,t2} = BLV_{UT,t1}$ then stop. IRR=i. Else adjust i based upon the numerical analysis routine and go to step 2.

An interesting way to think about this calculation is to consider the cash flow stream of the treated regime minus the cash flow stream of the untreated regime. This cash flow stream is the marginal investment cash flow stream in perpetuity and is used in the methodology to calculate the internal rate of return for the marginal investment of interest. As with any IRR calculation, several problems can occur based upon the timing and sign of the cash flows. In some instances no real IRR may exist and in other situations multiple IRRs may exist for unconventional cash flow streams (see, for example Clutter et al, 1982).

Now consider an existing stand at age A where some additional set of treatments is contemplated. We assume that when the stand is clearfelled then it is valued at BLV_F (future) – whether it is treated or not.

The NPV of the untreated regime can be calculated as:

$$NPV_{UT,t1} = \frac{\sum_{j=A}^{t1} (D_{UT,j} - C_{UT,j})(1+i)^{t1-j} + BLV_F}{(1+i)^{t1-A}} \tag{1.13}$$

The NPV of the untreated regime can be calculated as:

$$NPV_{T,t2} = \frac{\sum_{j=A}^{t2} (D_{T,j} - C_{T,j})(1+i)^{t2-j} + BLV_F}{(1+i)^{t2-A}} \tag{1.14}$$

Again to solve for the marginal IRR equations 1.13 is subtracted from 1.14 and solved for i. The technique described above should be used to calculate the IRR. This calculation is somewhat more complicated due to the need for both the current and future costs, revenues, and optimal future rotations.

This technique can easily be added into a growth and yield simulator that has a financial analysis component to compute the marginal IRR of some additional set of silviculture treatments. Similarly, output from a yield simulator can be used to develop a spreadsheet that calculates these internal rates of return. This technique outlines a financially defensible method for calculating marginal IRR associated with additional silviculture investments.

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