Development and Applications of the Relative Spacing Models for Loblolly Pine Plantations

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EXECUTIVE SUMMARY

The development through time of relative spacing was modeled for loblolly pine (*Pinus taeda* L.) plantations across the Lower Coastal Plain (LCP) and Piedmont and Upper Coastal Plain (PUCP) regions of the southern United States, using data from Coastal Plain Culture/Density study and SAGS Culture/Density Study. The lower asymptotic limit of relative spacing decreases exponentially with the increasing planting density in both the LCP and PUCP regions. It is also correlated with site index in the LCP region, but not in the PUCP region. In the PUCP region, loblolly pine plantation stands with the intensive management regime approach a higher value of minimum relative spacing than those with the operational management regime. In the LCP region, however, for a given planting density and site index loblolly pine plantation stands approach the same limit of relative spacing, regardless of management regime. Site index, planting density, and management intensity also affected other model parameters. How the resulting relative spacing models are used to determine thinning schedules and indirectly derive the survival patterns over time for young loblolly pine plantations were explored and outlined.

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1 INTRODUCTION

Relative spacing (RS, also known as Hart or Hart-Becking index) is defined as the ratio of the average distance between trees to the average dominant height of the stand. With square spacing the ratio is described as $RS = \sqrt{43,560 / N} / H_D$, where N is the number of trees per acre and H_D is average dominant height (ft). If triangular spacing is assumed, triangular-spacing RS value is a constant proportion (1.0745) of the corresponding square-spacing RS value. RS includes the number of trees and incorporates both site quality and age through dominant height; thus, it is a useful measure of stand density for developing thinning specifications for managed plantations, as proposed by Wilson (1946).

The trend of relative spacing over time is dependent on the relationship between height increment and mortality (Parker, 1978). Prior to the onset of competition-induced mortality, changes in relative spacing are due primarily to height growth. With crown closure, the increasing competition-induced mortality plays a more important role as changes in relative spacing are slowing down. Stands reach a minimum relative spacing value when the percentage height increment rate equals one-half of the percentage mortality rate (Parker, 1978), and then maintain it. It is believed that all stands of a given species approach a common minimum relative spacing asymptote with time, regardless of site quality and initial age (Clutter et al., 1983). This statement has rarely been tested with experimental studies, except for the study of Bredenkamp and Burkhart (1990) in which they found the minimum value of relative spacing varied with the planting density for *Eucalyptus grandis* plantations. Parker (1978) found significant correlation between the lower asymptotic limit of relative spacing and site index in radiata pine (*Pinus radiata* D. Don) unthinned plantations in New Zealand, but not in loblolly pine unthinned natural stands or in slash pine (*Pinus elliottii* Engelm.) unthinned plantations and natural stands in the southeastern United States. For plantation stands, the initiation of intraspecific competition and the lower asymptotic limit of relative spacing vary with site quality, stocking and other factors. Thus, the trajectories of relative spacing over time are also affected by these factors. However, a typical inverse-J trend of the relative spacing and age relationship is universal to systems of measurement and independent of species (Bredenkamp and Burkhart, 1990).

Most mortality and survival models have been developed for stands where mortality is density-dependent (Amateis et al., 1997; Zhao et al., 2007). Therefore, it is not reasonable to apply these mortality models for young stands where mortality is not expected to be density-dependent. Especially during the first several years after planting, mortality will depend on the quality of seedlings, stand establishment practices, characteristics of the sites, and some stochastic elements such as insect or disease attacks. If survival models are developed directly from survival data for these young stands, difficulties can occur in fitting models as well as model selection (Woollons, 1998; Zhao et al., 2007). Even with a two-step modeling strategy, the resulting models still have difficulties in capturing the dynamics of stand survival in early stages of stand development. Based on the changes in average spacing with the height of the stand, Beekhuis (1966) proposed a mortality model and then García (1981) developed an approximation for this model. Parker (1978) also noted that a combination of a height growth curve and the relative spacing time line implicitly defines survival curves. We propose that together with a relative spacing trend model and a dominant height model, García's mortality model can be used to indirectly describe survival trajectories over time for young stands.

With data from two loblolly pine culture and density studies, the objectives of the present study are: (1) to develop relative spacing time trend models; (2) to investigate how planting density, site quality, physiographic region, and management intensity level affect the development of relative spacing over time, especially the limiting value for relative spacing; and finally (3) to explore how resulting relative spacing models can be used for scheduling thinning and indirectly modeling the mortality for young loblolly pine plantations.

2 MATERIALS AND METHODS

2.1 Study Installations

Data for the analyses described in this study were obtained from plots of the Plantation Management Research Cooperative's (PMRC) Coastal Plain Culture/Density Study and SAGS (South Atlantic Gulf Slope) Culture/Density Study. The objectives of these studies are to quantify the effects of intensive silviculture and current operational practices on the growth and yield of loblolly pine plantations across a wide range of densities and to investigate potential interactions between cultural intensity and stand density across broad soil categories. The Coastal Plain Culture/Density study was established in 1995/96. Seventeen installations were established in the Lower Coastal Plain (LCP) of Georgia, Florida and South Carolina, across five CRIFF soil groups. At age 12, 14 loblolly installations remained in this study. The SAGS Culture/Density study was established in 1997/98. Twenty-three installations were established in the Piedmont and Upper Coastal Plain (PUCP) regions of Georgia, Alabama, Florida, Mississippi and South Carolina. This study was stratified over seven broad soil classes. After age 10, all 23 installations remained in this study.

In both culture/density studies, site preparation and subsequent silvicultural treatments were designed to represent two levels of management intensity: operational and intensive culture (Table 1). Within both the intensive and operational treatments, six loblolly pine subplots with densities of 300, 600, 900, 1200, 1500 and 1800 trees per acre were randomly located and established in each installation. To ensure the targeted initial density, each planting spot was double-planted and reduced to a single surviving seedling after the first growing season. For detailed information on these two studies such as soils and treatments carried out for each management level, refer to Harrison and Kane (2008) and Zhao et al. (2008), respectively.

Table 1. Silvicultural treatments for the Culture/Density study in the Lower Coastal Plain (LCP) and Piedmont/Upper Coastal Plain (PUCP)

LCP C/D Study		PUCP C/D Study	
Operational Treatment	Intensive Treatment	Operational Treatment	Intensive Treatment
Bedding	Bedding	Tillage including subsoiling on some sites	Tillage including subsoiling on some sites
Fall banded chemical site preparation	Fall broadcast chemical site preparation	Broadcast chemical site preparation	Broadcast chemical site preparation
	Tip moth control		Tip moth control
Herbaceous weed control: 1st year banded	Repeated herbicide application to achieve complete vegetation control	Hardwood control: 1st year banded	Repeated herbicide application to achieve complete vegetation control
Fertilization: At planting, 500 lbs of 10-10-10; Before 8th growth season, 200 lbs N + 25 lbs P	Fertilization: At planting, 500 lbs of 10-10-10; Spring 3rd grow season, 600 lbs 10-10-10 + micronutrients + 117 lbs NH_4NO_3; Spring 4th grow season 117 lbs NH_4NO_3; Spring 6th grow season 300 lbs NH_4NO_3; Spring 8th and 10th grow season 200 lbs N + 25 lbs P	Fertilization: At planting, 500 lbs of 10-10-10; Before 8th growth season, 200 lbs N + 25 lbs P	Fertilization: At planting, 500 lbs of 10-10-10; Spring 3rd grow season, 600 lbs 10-10-10 + micronutrients + 117 lbs NH_4NO_3; Spring 4th grow season 117 lbs NH_4NO_3; Spring 6th grow season 300 lbs NH_4NO_3; Spring 8th grow season 200 lbs N + 25 lbs P

2.2 Measurements

Dormant-season tree measurements were collected at ages 2, 4, 6, 8 and 10 years for both culture/density studies and at age 12 years for the culture/density study in the LCP. After the fourth growing season, diameters of all trees (DBH) were measured. Total height (H) and height to live crown were measured on every other tree. Total heights of unmeasured trees were estimated from the model $\ln(H) = b_0 + b_1 DBH^{-1}$ separately fitted for height measured trees at each measurement in each plot.

The average dominant height (H_D) is defined as the average height of trees with diameter (DBH) larger than the average DBH of the stand. Base age 25 years site index values were estimated for each installation using the dominant height of the plot with the planting density of 1483 trees ha^{-1} and operational treatments at the age of the most recent measurement. Site index was calculated using the site index equations developed by Borders et al. (2004) for second rotation loblolly pine plantations:

$$S = \alpha_0 \left\{ 1 - \left[1 - \left(\frac{H_D}{\alpha_0} \right)^{1/\alpha_1} \right]^{25/A} \right\}^{\alpha_1} \quad (1)$$

where $\alpha_0 = 117.6$ and $\alpha_1 = 1.336527$ in the PUCP, $\alpha_0 = 136.6$ and $\alpha_1 = 1.202941$ in the LCP, using imperial units. Site indices ranged from 73.5 to 92.3 feet for the study in the PUCP, and from 74.8 to 102.6 ft for the study in the LCP.

Square-spacing relative spacing is calculated for each plot at each measured age. The RS value ranged from 0.098 to 0.284 with a mean of 0.152 and standard deviation of 0.044 for the LCP culture/density study at age 12, and ranged from 0.107 to 0.359 with the mean of 0.187 and standard deviation of 0.057 for the PUCP culture/density study at age 10.

2.3 Model Development

Based on characteristics of the relative spacing and graphic examination of the relative spacing change over time, the following differential equation to model the trend of relative spacing toward a lower asymptotic limit was used:

$$\frac{dRS}{dA} = \delta RS\ A^{-1}[\phi - \ln(RS)] \tag{2}$$

where *RS* is relative spacing and *A* is age; δ and ϕ are the parameters. This differential equation is a modification of an equation form originally suggested by Clutter (1963) for basal area growth. The solution of the differential Eq. (2) can be written:

$$RS = \alpha \exp(\beta A^{-\delta}), \tag{3}$$

where $\alpha = e^{\phi}$ defines the lower asymptotic limit, $\beta = e^{C}$ is constant of integration and δ is a shape parameter.

Preliminary analysis indicated that all terms in Eq. (3) varied with planting density (N_0, trees per acre), site index (*SI*, ft), and/or management intensity (*TRT*, a dummy variable indicating the management intensity: $TRT = 1$ for intensive culture and $TRT = 0$ for operational culture). Therefore, Eq. (3) was fitted to each of the data sets from the LCP and PUCP culture/density studies, and was used to explore the relationships of the lower asymptotic limit, rate and shape terms with these variables. Examination of the relationship between the lower asymptotic limit and planting density revealed an inverse J-shaped relationship, and the lower asymptotic limit was also affected linearly by site index and/or management intensity. Both rate and shape terms were affected linearly by planting density, site index, and/or management intensity. Thus, these terms were generalized as:

$$\alpha = \alpha_0 + \alpha_1 TRT + \alpha_2 SI + \alpha_3 (N_0 / 100)^{-\gamma}$$
$$\beta = \beta_0 + \beta_1 TRT + \beta_2 SI + \beta_3 (N_0 / 100) \tag{4}$$
$$\delta = \delta_0 + \delta_1 TRT + \delta_2 SI + \delta_3 (N_0 / 100)$$

Combining Eqs (3) and (4) resulted in a general full model to describe the development through time of relative spacing values. This full model was fitted for the LCP and PUCP culture/density studies, respectively.

3 RESULTS

Parameter estimates for the relative spacing development model for each of the two culture density studies are given in Table 2. Variables whose associated parameter estimates were not statistically significantly different from zero are excluded in the general model form. The fitted models for the relative spacing development over time were

$$RS = [0.0080TRT + 0.2541(\tfrac{N_0}{100})^{-0.5425}]\exp\{(6.4041 \\ -1.0879TRT - 0.00049N_0)A^{-(0.1288+0.0095SI-0.00008N_0)}\} \tag{5}$$

for the PUCP, where $R^2 = 0.907$ and $MSE = 0.0024$; and

$$RS = [0.0008SI + 0.2635(\tfrac{N_0}{100})^{-0.9217}] \\ \exp\{(-14.0415 - 1.3679TRT + 0.2432SI \\ + 0.0011N_0)A^{-(-1.8491+0.0331SI+0.0002N_0)}\} \tag{6}$$

for the LCP, where $R^2 = 0.934$ and $MSE = 0.0012$.

Loblolly pine plantations with different conditions asymptotically approach different limits of relative spacing (Figure 1). The lower asymptotic limit varies with planting density and management intensity in the PUCP, and with planting density and site index in the LCP. The lower asymptotic limit decreases exponentially with increasing planting density in both the LCP and PUCP regions. In the PUCP region, loblolly pine plantations with the intensive management regime will approach higher minimum relative spacing than with the operational management regime, and the lower asymptotic limit of relative spacing is not correlated with site index. In the LCP region, however, the lower asymptotic limit of relative spacing increases linearly with increasing value of site index. For a given planting density and site index, loblolly pine plantations in the LCP will approach the same limit of relative spacing, regardless of management intensity level. In general, for a given planting density, the lower asymptotic limit of relative spacing for loblolly pine plantations is higher in the LCP than in the PUCP.

Table 2 Table **1. Silvicultural treatments for the Culture/Density study in the Lower Coastal Plain (LCP) and Piedmont/Upper Coastal Plain (PUCP)**

Parameter	Piedmont/Upper Coastal Plain			Lower Coastal Plain		
	Estimate	Standard error	p-value	Estimate	Standard error	p-value
$\hat{\alpha}_1$	0.0080	0.0025	0.0015			
$\hat{\alpha}_2$				0.0008	0.0001	<0.0001
$\hat{\alpha}_3$	0.2541	0.0300	<0.0001	0.2635	0.0205	<0.0001
$\hat{\gamma}$	0.5425	0.0280	<0.0001	0.9217	0.1217	<0.0001
$\hat{\beta}_0$	6.4041	0.3624	<0.0001	-14.0415	1.6778	<0.0001
$\hat{\beta}_1$	-1.0879	0.1399	<0.0001	-1.3679	0.1301	<0.0001
$\hat{\beta}_2$				0.2432	0.0236	<0.0001
$\hat{\beta}_3$	-0.0489	0.0209	0.0196	0.1129	0.0304	0.0002
$\hat{\delta}_0$	0.1286	0.0465	0.0057	-1.8491	0.1274	<0.0001
$\hat{\delta}_2$	0.0095	0.0009	<0.0001	0.0331	0.0016	<0.0001
$\hat{\delta}_3$	-0.0081	0.0032	0.0111	0.0217	0.0042	<0.0001

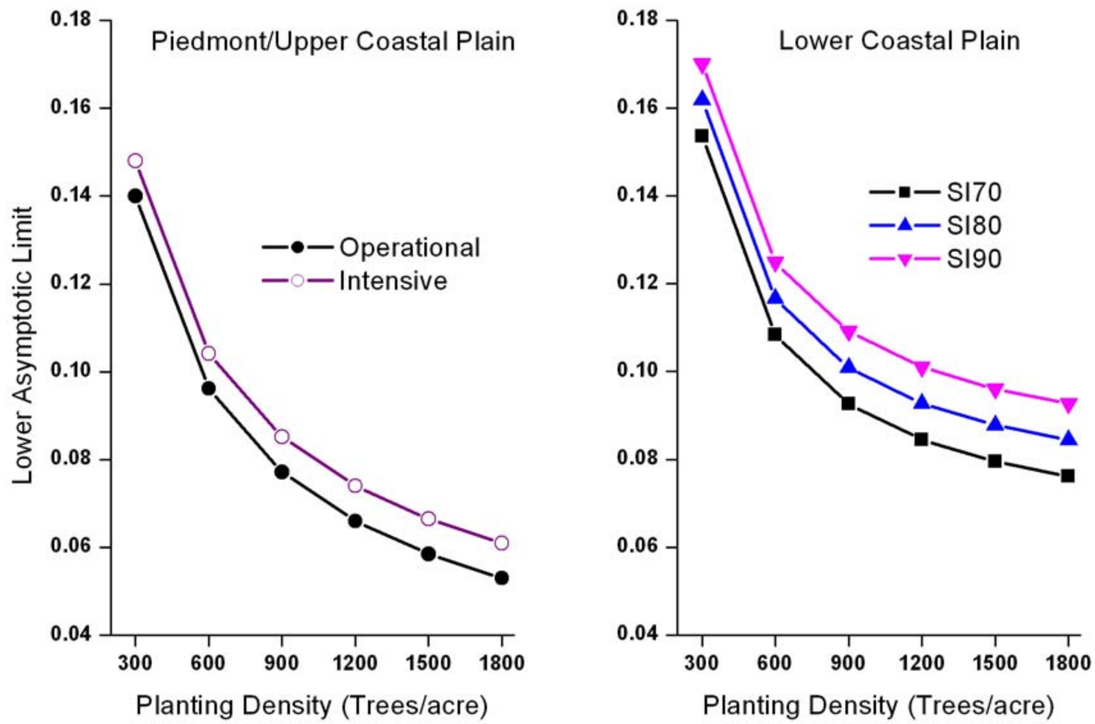

Figure 1. The lower asymptotic limit of relative spacing for loblolly pine plantations with different planting density and management intensity (Intensive and Operational) in the Piedmont/Upper Coastal Plain, and with different planting density and site index in the Lower Coastal Plain.

The planting density not only affects the asymptotic limit of relative spacing, but also affects other parameters of relative spacing trend over time. The effects of planting density on the rate and shape parameters were significant in both the LCP and PUCP. However, planting density affected both terms in an opposite way for these two regions. With increasing planting density, both the rate and shape terms decreased in the PUCP but increased in the LCP.

Both the rate and shape parameters in the relative spacing model for loblolly pine plantation in the LCP were correlated with site index; in the PUCP only the shape parameter was related to site index. These parameters increased with increasing site index, for a given planting density. The management intensity significantly affected the rate parameter in the LCP and the PUCP. Thus, the development through time of relative spacing for loblolly pine plantations is determined jointly by the planting density, site index, and management intensity (Figure 2 and Figure 3).

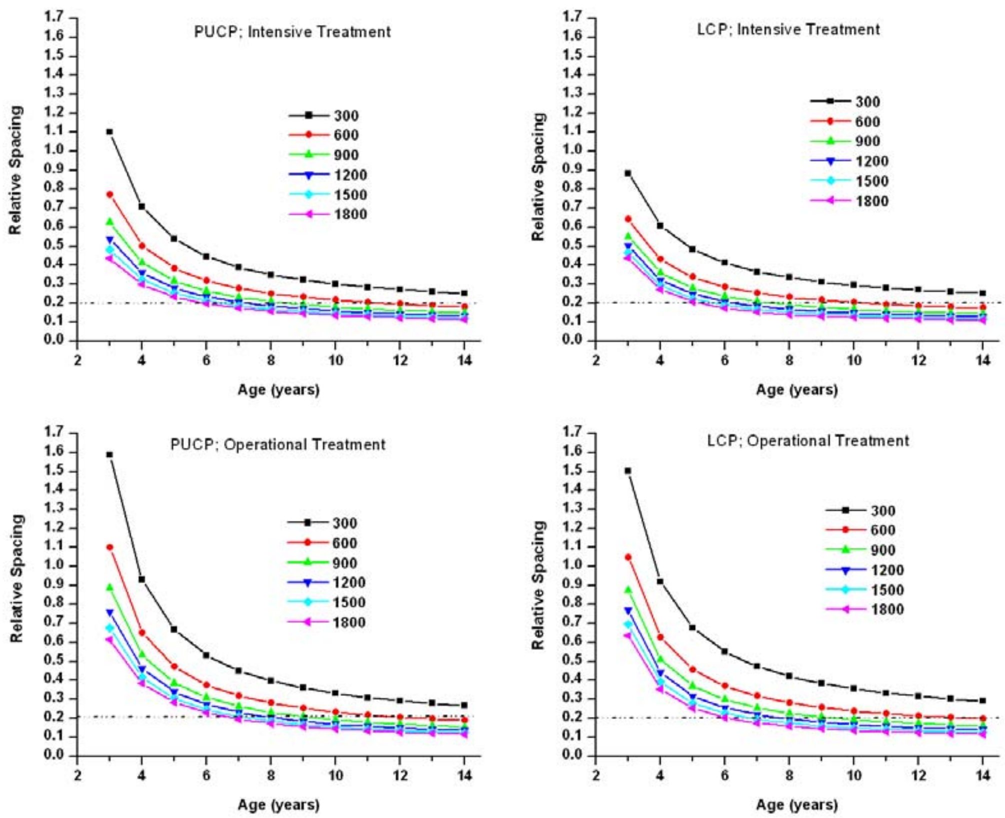

Figure 2. Relative spacing trends over time for loblolly pine stands with six levels of planting density, two levels of management intensity (Intensive and Operational), and site index 80 ft. Predictions are for the Piedmont/Upper Coastal Plain (PUCP) using Model (5) and the Lower Coastal Plain (LCP) using Model (6).

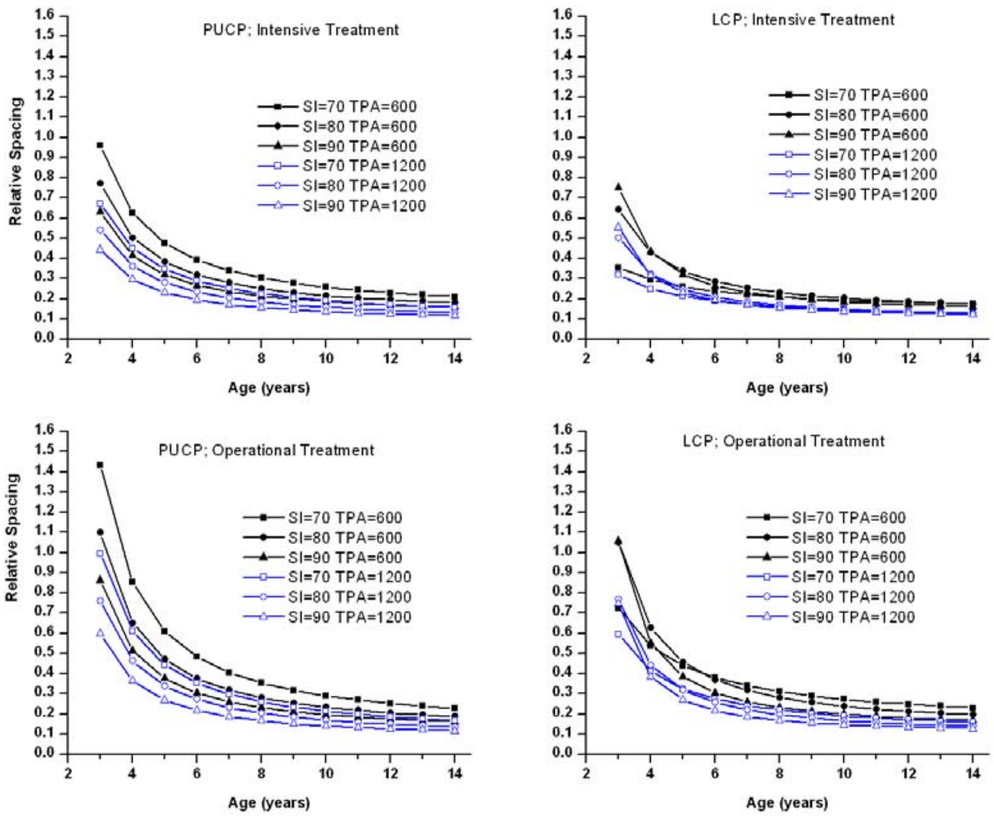

Figure 3. Relative spacing trends over time for loblolly pine stands with two levels of planting density (600 and 1200 trees/acre), three levels of site index (70, 80, and 90 ft), and two levels of management intensity (Intensive and Operational). Predictions are for the Piedmont/Upper Coastal Plain (PUCP) using Model (5) and the Lower Coastal Plain (LCP) using Model (6).

4 MODEL APPLICATIONS

4.1 Thinning Schedules

Since relative spacing is a function of number of trees per acre and stand dominant height, maximum stocking limits associated with the lower asymptotic limit of the relative spacing value ($\hat{\alpha}$) that can be expected in unthinned plantations at any age (*A*) and site index (*SI*) can be approximated by:

$$\hat{N}_{\max\ (A,SI)} = 43,560/(\hat{\alpha} \cdot \hat{H}_{D\ (A,SI)})^2 \quad (7)$$

where $\hat{N}_{\max\ (A,S)}$ is the maximum stocking limit in number of trees per acre at any age (*A*) on site index (*SI*), and $\hat{H}_{D\ (A,SI)}$ is stand dominant height at any age (*A*) on site index (*SI*).

Thinning schedules can be determined by setting proper upper and lower bounds of relative spacing. The lower bound of relative spacing is established to avoid density-related mortality, which determines when to thin. Stands need to be thinned before they reach the lower bound of relative spacing. The principal objective in setting the upper bound of relative spacing is to maintain adequate site occupancy; post-thinning relative spacing should not exceed the upper bound. The distance between the upper and lower relative spacing bounds determines the potential thinning yield.

For a given lower bound of relative spacing RS_L, the age at which stands reach this point and need to be thinned A_T is calculated by:

$$A_T = [\hat{\beta} / \ln(RS_L / \hat{\alpha})]^{1/\hat{\delta}}. \quad (8)$$

Assuming that a potential thinning criterion is set at 0.2 of relative spacing, the estimated ages at which plantations with different conditions meet this criterion are given in Table 3. Obviously, the expected age depends on planting density, site quality, management intensity, and physiographic region.

Given an upper bound of relative spacing (RS_U) to be kept after thinning, the number of trees per acre in the residual stand ($N_{A,SI}$) is

$$N_{A,SI} = 43,560/(RS_U \cdot H_{D\ (A,SI)})^2 \quad (9)$$

where $H_{D\ (A,SI)}$ is stand dominant height at any age (*A*) on site index (*SI*).

Table 3. Estimated ages when plantations meet a potential thinning criterion of 0.2 relative spacing by physiographic region, planting density, management intensity, and site quality.

Region	Planting Density	Operational			Intensive		
		SI = 70	SI = 80	SI = 90	SI = 70	SI = 80	SI = 90
Piedmont/ Upper Coastal Plain	300	41.9	27.8	20.0	40.7	27.1	19.6
	600	17.4	12.6	9.7	15.6	11.4	8.9
	900	12.9	9.5	7.5	11.3	8.5	6.8
	1200	10.9	8.2	6.5	9.5	7.2	5.8
	1500	9.8	7.4	5.9	8.4	6.5	5.3
	1800	9.1	6.9	5.5	7.7	5.9	4.9
Lower Coastal Plain	300	115.7	45.7	26.6	42.7	33.4	22.9
	600	19.9	13.6	10.0	9.1	10.3	8.7
	900	12.0	9.5	7.6	6.4	7.5	6.7
	1200	9.2	7.8	6.5	5.8	6.3	5.8
	1500	7.8	6.8	5.8	5.0	5.6	5.2
	1800	6.8	6.1	5.4	4.7	5.2	4.9

4.2 Mortality Prediction

Mortality models for young loblolly pine plantations were derived indirectly from relative spacing trends over time and dominant height curves, rather than directly from survival data. The average square spacing (*S*) is defined as $S = \sqrt{43,560/N}$ for *S* in feet and *N* in the number of trees per acre. García (1981) found that the slope of the average spacing (*S*) and dominant height (H_D) curves depends only on the relative spacing $RS = S/H_D$, increasing from 0 to α as *RS* decreases to the limiting relative spacing α. Therefore, he proposed the following model to approximate Beekhuis's model (Beekhuis, 1966):

$$S_2^{\gamma} - S_1^{\gamma} = (\alpha H_{D2})^{\gamma} - (\alpha H_{D1})^{\gamma}, \qquad (10)$$

where (H_{D1}, S_1) and (H_{D2}, S_2) are consecutive data points of dominant height and average spacing. The exponent γ, depending on the limiting relative spacing α and a value of relative spacing (*a*) at which density-related mortality begins, is calculated using García (1981):

$$\begin{aligned} \gamma &= \ln 2 / \ln[1 + r - \sqrt{r^2 - (1/\alpha - 1/a)^2 / 100}] \\ r &= \frac{1 - \alpha / a}{\sqrt{1 + (10\alpha)^2} - 1} \end{aligned} \tag{11}$$

With different values of *a* and α, Beekhuis's model and García's approximation model (10) have been used in growth models for radiata pine plantations ($a = 0.30,\ \alpha = 0.11$, i.e. $\gamma = 5.5$ and $\alpha = 0.11$ for triangular spacing) and Douglas-fir (*Pseudotsuga menzeisii*) ($a = 0.18,\ \alpha = 0.09$, i.e. $\gamma = 6.6$ and $\alpha = 0.09$ for triangular spacing) in New Zealand (García 1981, 2009).

An alternative form of Eq. (10) is:

$$N_2 = N_1 \left[1 - (\alpha H_{D1} \sqrt{N_1 / 43{,}560})^{\gamma} + (\alpha H_{D2} \sqrt{N_1 / 43{,}560})^{\gamma} \right]^{-2/\gamma}. \tag{12}$$

If the number of surviving trees per acre is predicted from planting, that is, $N_0 = N_1$ is planting density and $H_{D1} = 0$, then the number of trees when stand dominant height reaches any H_D is estimated with

$$N = N_0 \left[1 + (\alpha H_D \sqrt{N_0 / 43{,}560})^{\gamma} \right]^{-2/\gamma}. \tag{13}$$

Models (12) or (13) describe the survival patterns of plantation stands over height. Combined with the dominant height model $H_D = f(SI,\ Age)$, these models can easily be used to derive the survival patterns over time.

In the previous applications of Beekhuis's model and García's 1981 approximation model, the lower asymptotic limit of relative spacing was assumed to be a constant for a specific tree species (Beekuis, 1966; García, 1981). Our results indicated that the asymptotic limit of relative spacing for loblolly pine plantations varied with initial density, physiographic region, and /or site quality, or management intensity. Therefore, the assumption of a constant limiting value for relative spacing is not reasonable. In the present application of Models (12) and (13) to project the number of surviving trees throughout the development of loblolly pine plantations, the limiting values of relative spacing α are directly estimated from different stand conditions, rather than a predetermined value. Thus, both parameters α and γ in Model (12) or (13) are flexible, varying with the stand conditions. García (2009) noticed the usefulness of the added flexibility of γ, one of three parameters in his 2009 model, and mentioned further research on this topic might be interesting.

For loblolly pine plantations with initial conditions of specific site index (*SI*, ft) and planting density (N_0, trees/acre), the lower asymptotic limits of relative spacing were estimated, using the lower asymptotic limit term in the relative spacing model developed in the current study, that is

$$\alpha = 0.008TRT + 0.2541(N_0/100)^{-0.5425} \tag{14}$$

for the PUCP, and

$$\alpha = 0.0008SI + 0.2635(N_0/100)^{-0.9217} \tag{15}$$

for the LCP. Assuming a free-growing relative spacing value of 0.5, that is $a = 0.5$, the parameter γ in Model (13) was calculated using the formula (11) with the estimated values of lower asymptotic limit for relative spacing. The dominant heights (H_D) at any age (A) on site index (SI) were projected with the models developed by Borders et al. (2004) for loblolly pine plantations:

$$H_D = \alpha_0 \left\{ 1 - \left[1 - \left(\frac{SI}{\alpha_0} \right)^{1/\alpha_1} \right]^{A/25} \right\}^{\alpha_1} \quad (16)$$

where the estimates of parameters α_0 and α_1 are same as given in Model (1). Finally, the surviving trees were predicted with Model (13). The survival trajectories over time for the hypothetical stands are shown in Figure 4.

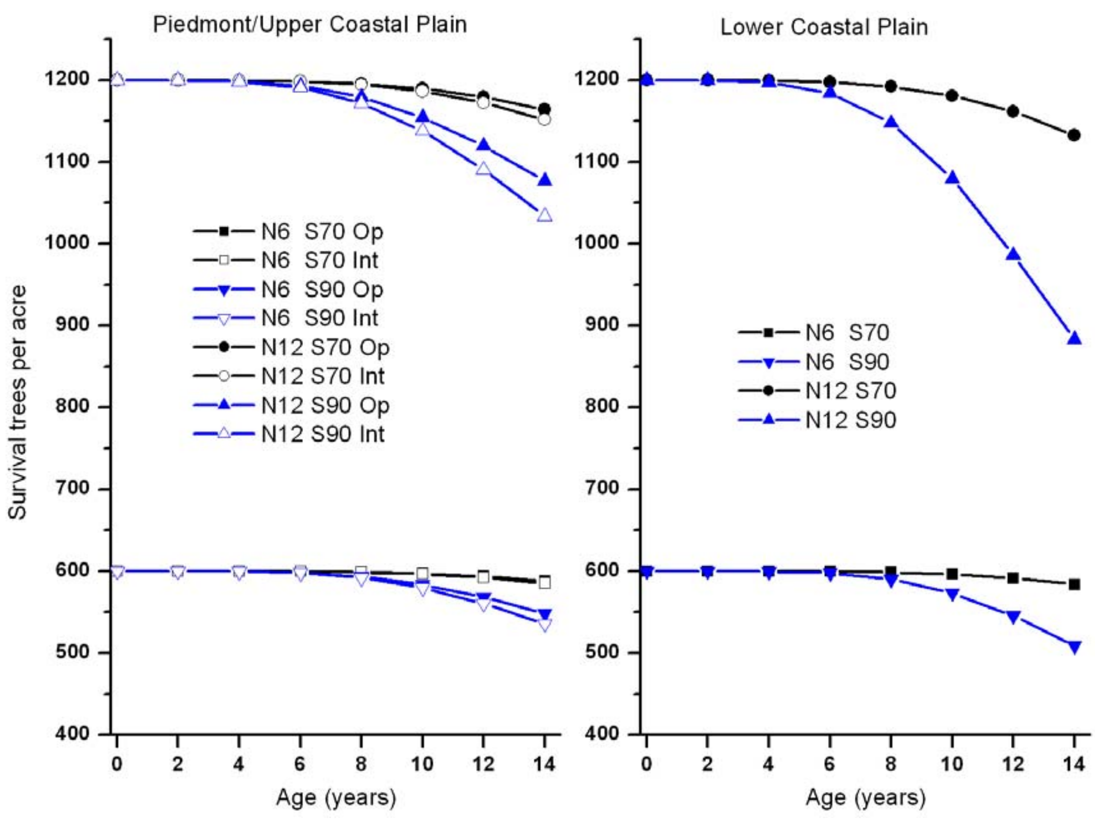

Figure 4. Survival projections for loblolly pine plantations with planting density of 600 and 1200 trees/acre and site index of 70 and 90 ft, by physiographic regions.

For given initial conditions ($N_1,\ H_{D1},\ A_1$), the dominant height H_{D2} at age A_2 could be projected with the projection model (Borders et al. 2004):

$$H_{D2} = \alpha_0 \left\{ 1 - \left[1 - \left(\frac{H_{D1}}{\alpha_0} \right)^{1/\alpha_1} \right]^{A_2/A_1} \right\}^{\alpha_1} \tag{17}$$

where the estimates of parameters α_0 and α_1 are same as given in Model (1). For given initial conditions ($N_1,\ SI,\ A_1$), the dominant height H_{D2} at age A_2 and H_{D1} at age A_1 could be projected with Model (16). Then, the number of surviving trees at age A_2 could be predicted with Model (12), in which the limiting value of relative spacing α is estimated with equations (14) or (15) and parameter γ is calculated using formula (11).

In order to evaluate its suitability, this approach was used for data of our culture and density studies to predict the survival at ages 4, 6, 8, 10 and 12 using stand conditions two years prior to prediction age. A percent residual number of surviving trees per hectare (100 × (observed survival – predicted survival) / observed survival) was computed for each two-year interval observation. The mean percent residual number of trees per hectare by physiographic region and planting density, as well as by prediction interval, is shown in Table 4.

Table 4. Mean percent residual (100 × (observed survival – predicted survival) / observed survival) for two-year interval survival observations from ages 2 to 10 or 12 years by physiographic region, planning density, and prediction interval.

Region	By planting density				By prediction interval			
	Density	Mean	SD	N	Interval	Mean	SD	N
Lower Coastal Plain	300	-0.5	2.6	111	2 - 4	-0.3	1.0	43
	600	-1.5	7.6	115	4 - 6	-0.2	9.1	154
	900	-0.8	6.5	117	6 - 8	0.9	4.3	158
	1200	0.3	7.4	115	8 - 10	-1.6	8.5	168
	1500	-1.0	14.7	116	10 - 12	-0.9	13.0	165
	1800	0.7	10.5	114				

Piedmont/ Upper Coastal Plain	300	-1.2	3.6	181	2 - 4	-0.4	1.5	265
	600	-1.2	3.5	180	4 - 6	-0.5	1.2	257
	900	-1.5	3.1	175	6 - 8	-2.8	5.4	276
	1200	-1.9	3.6	174	8- 10	-3.4	5.1	274
	1500	-2.3	5.0	182				
	1800	-2.9	5.2	180				

5 DISCUSSION AND CONCLUSIONS

Relative spacing is a comprehensive index that involves the number of survival trees per unit area and stand average dominant height. Spacing trial studies including our two loblolly pine culture/density studies that include a wide range of planting density have identified density effects on the average height and dominant height for loblolly pine (Sharma et al., 2002; Harrison and Kane, 2008; Zhao et al., 2008; Antón-Fernándeź et al., 2009). Height differences due to initial planting density are significant from age 6 (Zhao et al., 2008; Antón-Fernándeź et al., 2009). The differences between dominant heights at different planting densities are maintained and do not tend to disappear with age (Antón-Fernándeź et al., 2009), which implies different dominant heights at site index reference age. Our culture/density studies and other studies have also shown that the intensive management regimes including complete vegetation control and repeated fertilization increase long-term dominant height growth and pine production, compared with the operational management regimes (Zhao et al., 2008; Zhao et al., 2009a; Zhao et al. 2009b). Moreover, initial planting density and management intensity also affect stand mortality (Zhao et al., 2007; Zhao et al., 2008). In the present study we assigned one site index value to all plots at each installation using the plot of 1483 trees/ha planting density and operational management regime, trying to separate the effect of site quality from the effects of initial planting density and management intensity on relative spacing development over time.

The relative spacing changes over time for loblolly pine plantations are developed separately for the PUCP and LCP physiographic regions of the southern United States. The estimation of lower asymptotic limit of relative spacing indicated that the lower asymptotic limit for loblolly pine plantations in the LCP increased linearly with the increasing site index. Given an initial density, loblolly pine plantations in the LCP on better sites will approach a higher minimum relative spacing. This implies that a lower maximum stocking limit may be expected, and higher mortality is related to better productivity for unthinned loblolly plantations in the LCP. For loblolly pine plantations in the PUCP, there was no significant relationship between the lower asymptotic limit of relative spacing and site index. When examining the lower asymptotic limit of relative spacing for loblolly pine natural stands, slash pine natural stands, and slash pine plantations in the southeastern United States, as well as radiata pine plantations in New Zealand, Parker (1978) found a significant relationship between the lower asymptotic limit and site index in only radiata pine plantations. The lower asymptotic limit for radiata pine plantations decreased linearly with the increasing site index, and it was 0.053 for the average observed site index.

For a given initial density, loblolly pine plantation stands in the LCP will asymptotically approach higher value of minimum relative spacing than that in the PCUP (Figure 1). Our estimates of the lower asymptotic limit ranged from 0.053 to 0.148 for loblolly pine plantations in the PUCP, and ranged from 0.076 to 0.170 in the LCP. This may be explained by the facts that for a given initial density and site index loblolly pine plantation stands in the LCP usually have high mortality rate than in the PUCP (Zhao et al., 2007), and in general loblolly pine plantations in the LCP are more productive. For natural loblolly pine stands, Parker (1978) also found the lower asymptotic limit of relative spacing varied with physiographic region. Loblolly pine natural stands in the

Coastal Plain and the Piedmont physiographic provinces approached to 0.165 and 0.183 of lower asymptotic limit of relative spacing, respectively.

These two experimental culture/density studies presented here cover a wide range of planting density, from 741 to 4448 trees per hectare. This is helpful for the investigation of the relationship between the lower asymptotic limit of relative spacing and initial density. Given a site quality, the lower asymptotic limit decreases exponentially with the increasing planting density for loblolly pine plantations in both the LCP and PUCP regions. The same inverse J-shaped relationship between the lower asymptotic limit spacing and initial density was also observed in *Eucalyptus grandis* plantations (Bredenkamp and Burkhart, 1990). Plantation stands with different planting densities will asymptotically approach different lower limits of relative spacing rather than a common one. This is explained by the difference in self-thinning for these stands. Because stands established at different initial densities do not thin to the same final density (Reynolds and Ford, 2005), they do not approach to the same lower limit of relative spacing. Plantation stands established at high and low densities self-thin in different ways (Turnblom and Burk, 2000), thus have different relative spacing trends over time.

In the PUCP region, loblolly pine plantation stands with the intensive management regime approach a higher value of minimum relative spacing than those with the operational management regime. For loblolly pine plantations in the LCP, however, the effect of management intensity on the lower limit of relative spacing is not detectable. It seems that for a given planting density and site index all loblolly pine plantation stands in the LCP region approach the same limit of relative spacing, regardless of management intensity level. The effect of management intensity on the shape parameter is still significant in both the relative spacing models developed for loblolly pine plantations in the PUCP and LCP regions. Before and shortly after crown closure loblolly pine plantation stands with the intensive management had lower values of relative spacing than operational stands with the same initial density, due to larger height with intensively managed stands. During this period, height growth rather than competition-induced mortality plays a more important role in relative spacing change. Previous studies indicated that sustained complete competition control affects long-term productivity of loblolly pine plantations in the PUCP region (Nilsson and Allen, 2003; Zhao et al., 2009a); repeated fertilization and complete competition control also have long-term effects on growth of the slash pine plantations in the LCP (Zhao et al., 2009b). Thus, it is understandable that plantation stands with the intensive management regimes including complete vegetation control and repeated fertilization treatments have different stand development patterns, compared with stands with the operational regime.

Our results obviously indicated that loblolly pine plantations with different conditions may asymptotically approach different minimum relative spacing values. This lower limit of relative spacing is related to physiographic region, site quality, planting density, and/or management intensity. This finding did not confirm the statement of Clutter et al. (1983) that "regardless of site quality and initial age, all stands of a given species seem to asymptotically approach a common minimum relative spacing value." It should be noticed that our relative spacing models are based on data with the limited age range. Although our conclusions make sense, it is still worthwhile to validate them with data sets encompassing the entire rotational period.

Relative spacing is a viable stand density control in determining thinning regimes (Wilson, 1946, 1979). According to Wilson's rules of thumb (1946, 1979), with square spacing, the proper relative spacing intervals are (0.16, 0.14) for spruce, (0.20, 0.16) for white pine, (0.24, 0.16) for red pine, and (0.25, 0.20) for Jack pine. For loblolly pine plantations, we may set the upper and lower bounds of relative spacing at 0.30 and 0.20, respectively. This interval remains to be tested with our on-going thinning study conducted in plots of the culture and density studies. Practically, the thinning criteria proposed based on relative spacing or other mensurational variables should be justified with economic and other considerations. Once the thinning criteria of relative spacing is determined, then when loblolly pine plantation stands should be thinned and how many trees should be kept in the residual stands can be calculated, with the parameters of our resulting relative spacing models. Generally, for a given planting density the need for a first thinning for loblolly pine plantations can be earlier on land with high site quality than on land with low site quality (Table 3). For the same site quality, the plantations with higher planting density meet the thinning criterion earlier than those with lower planting density. The intensive management regime improved the growth of loblolly pine plantations, thus plantations with intensive management need to be thinned earlier than those with the operational management regime.

Together with stand dominant height model, the resulting models can be used to indirectly derive the survival patterns over time for young loblolly pine plantations. Beekhuis's or García's model is determined by a lower asymptotic limit of relative spacing and a specific value of relative spacing at which density-related mortality occurs. In the present study, García's model form was used with a variable limiting value of relative spacing and a constant value of relative spacing at which density-related mortality occurs. The latter is set at 0.5, a free-growing value relative spacing that Pienaar (1977) set for slash pine. As the limiting value of relative spacing, the initiation of intraspecific competition may also vary with site quality, stocking and other factors. It is reasonable to add a variable value of relative spacing at which density-related mortality occurs in the model, although how to quantify this point remains an interesting research topic.

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